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Inelastic channels in the electroweak symmetry-breaking sector

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It has been argued that if light Higgs bosons do not exist then the self-interactions of W's become strong in the TeV region and can be observed in longitudinal WW scattering. We present a model with many inelastic channels in the WW scattering process, corresponding to the creation of heavy fermion pairs. The presence of these heavy fermions affects the elastic scattering of W's by propagating in loops, greatly reducing the amplitudes in some charge channels. Consequently, the symmetry-breaking sector cannot be fully explored by using, for example, the W^+W^+ mode alone; all $WW \to WW$ scattering modes must be measured.

If light Higgs bosons do not exist, it is believed that elastic longitudinal WW scattering will be enhanced, indicating the presence of new strong interactions at or above 1 TeV. (We use W to denote either the $W^\pm$ or Z^0 boson.) It has been claimed that the energy and luminosity of the SSC are large enough that, whatever form the new interactions take, they would be observable in WW two-body interactions via leptonic decays of W's [1]. This is called the "no-lose theorem." Study of the W^+W^+ mode has been particularly promoted [2] in the context of observing these strong interactions because the standard model background for this mode is small.

Recently, Chivukula and Golden have emphasized the possible existence of inelastic channels in the WW scattering process [3]. They studied an $O(4)\times O(n)$ model in which the WW interactions are almost entirely inelastic (to n species of pseudo-Goldstone bosons). Consequently, the elastic WW amplitude is reduced, the more so for a larger number n of inelastic channels. (The total event rate for elastic scattering, however, does not decrease as n increases [4].) In that model, inelastic scattering leads to a very broad low-energy resonance in the elastic WW scattering amplitudes. In this letter, we present a model with many inelastic channels in the electroweak sector and which has no resonances.

In general, the presence of inelastic channels corresponds to additional particles in the theory. Even if the production of these particles is not directly observed, they affect the *elastic* scattering of W's by propagating in loops. These loops necessarily contribute to the imaginary part of the elastic scattering amplitude, which is related by the optical theorem to the total cross section. The loops also contribute to the real part of the elastic amplitude, interfering with the Born contribution. This interference may dramatically reduce the signal in some charge channels, e.g. the W^+W^+ channel. Other channels may be enhanced, however, both by real and imaginary loop corrections. The model we present below has precisely this behavior. The moral is that to be certain of detecting the symmetry-breaking sector it will be necessary to measure scattering in all the final state WW modes.

The no-lose theorem, with its prediction of strong WW scattering in the absence of a light Higgs resonance, is based on the low-energy theorems for a theory with spontaneously-broken symmetry. The pattern of symmetry breaking for the electroweak sector is $SU(2)_L\times SU(2)_R \to SU(2)_V$ if we assume that it respects a custodial SU(2) symmetry. At energies $s \gg M_W^2$, the longitudinal vector bosons W correspond, via the equivalence theorem, to the three Goldstone bosons ϕ^a resulting from this broken symmetry. The interactions of these Goldstone bosons are described by low-energy theorems, which emerge automatically when we describe this broken symmetry using chiral lagrangians. In this approach, the Goldstone bosons are parametrized by the matrix

$$\Sigma = \exp\left(\frac{i\tau^a \phi^a}{f}\right), \quad a=1,2,3\,, \tag{1}$$

where τ^a are the Pauli matrices. The lowest energy term of the chiral lagrangian is

$$\mathscr{L} = \tfrac{1}{4} f^2 \, \mathrm{Tr}(\partial_\mu \Sigma \, \partial^\mu \Sigma^\dagger)\,. \tag{2}$$

It follows that the low-energy amplitude for $Z^0Z^0 \to W^+W^-$, for example, is

$$\mathscr{M}(Z^0Z^0 \to W^-W^+) = \frac{s}{f^2}\,, \tag{3}$$

where $f = 250$ GeV. This amplitude grows with increasing center-of-mass energy, becoming strong at around 1 TeV.

At higher energies, the amplitude (3) is modified by corrections of order $s^2/16\pi^2 f^4$ coming from higher-dimension operators induced by physics at a higher scale, and from Goldstone boson loop corrections [5]. Indeed, we know that the amplitude (3) must eventually break down, because it violates partial-wave unitarity ($|\mathrm{Re}\, a_0^0| > \frac{1}{2}$) at about $2\sqrt{2\pi} f \sim$ 1.2 TeV. Nevertheless, below the scale of unitarity breakdown, the scattering of W's will be roughly governed by eq. (3), as long as there are no "light" particles ($M \lesssim 1$ TeV) in the symmetry-breaking sector other than the Goldstone bosons. On the other hand, if there do exist such light particles, the amplitude (3) will only be valid for $s \ll M^2$.

If such light particles can be exchanged by the W's, as in the case of a light Higgs boson for example, there will be narrow resonances in the WW scattering amplitudes at $s \sim M^2$. We consider another possibility, particles with mass well below 1 TeV which can only be produced in pairs; in particular we have in mind heavy fermions. These fermions correspond to inelastic channels in the WW scattering process. Unlike exchange particles, they will not necessarily produce resonances in the elastic WW scattering amplitudes. They alter these amplitudes through loop effects, however, and lead to behavior markedly differently from that predicted by low-energy theorems at scales above the threshold for fermion pair production but below 1 TeV. (New physics must still enter at around 1 TeV, because the fermions, unlike the Higgs boson, do not unitarize the amplitudes.)

In this letter, we present a chiral lagrangian model coupled to heavy fermion doublets. This model has no resonances in the elastic WW scattering amplitudes, therefore we must study these amplitudes in the TeV region where they become strong, We find that the presence of the fermions dramatically reduces the scattering rates for the W^+W^+ and W^+Z^0 modes relative to the predictions of the low-energy theorems. On the other hand, the rates for scattering into the modes W^+W^- and Z^0Z^0 are enhanced. This enhancement partially results from the large imaginary part of the loop amplitude in the forward direction, which via the optical theorem is due to the large cross section. Here we will only present our results; details of the calculation will be given elsewhere [6].

The lagrangian for the model is

$$\begin{aligned}\mathscr{L} &= \tfrac{1}{4} N v^2 \, \mathrm{Tr}(\partial_\mu \Sigma \, \partial^\mu \Sigma^\dagger) \\ &\quad + \sum_{j=1}^{N} \left[\bar\psi^j i \partial\!\!\!/ \psi^j - g v (\bar\psi^j_L \Sigma \psi^j_R + \bar\psi^j_R \Sigma^\dagger \psi^j_L)\right], \\ \Sigma &= \exp\left(\frac{i\tau^a \phi^a}{\sqrt{N} v}\right), \\ \psi^j_L &= \tfrac{1}{2}(1-\gamma_5)\psi^j, \quad \psi^j_R = \tfrac{1}{2}(1+\gamma_5)\psi^j\,. \end{aligned} \tag{4}$$

The fields ψ^j represent N degenerate heavy fermion doublets with mass $m = gv$. The effects of the fermions on the WW scattering amplitudes will be significant when the Yukawa coupling g is large. To capture this, we will not calculate the amplitudes perturbatively in g, but rather in a $1/N$ expansion, holding the parameters g and v fixed as $N \to \infty$. The results will be valid for arbitrary Yukawa coupling g, i.e., for all values of the fermion mass m.

Were we to calculate perturbatively in the Yukawa coupling, the real part of the loop correction would contribute through interference with the tree-level amplitude, but the imaginary part would be higher order. In the large-N approach, the imaginary part of the loop correction contributes in leading order. Because it is related to the total cross section via the optical theorem, this contribution is important when there are many inelastic channels.

To leading order in $1/N$, the only corrections come from fermion loops. These contribute a divergence to the Goldstone boson self-energy. Accordingly, we add a counterterm

$$\delta\mathscr{L} = \delta Z (\tfrac{1}{4} N v^2) \, \mathrm{Tr}(\partial_\mu \Sigma \, \partial^\mu \Sigma^\dagger)\,, \tag{5}$$

with δZ chosen so that the residue of the Goldstone boson propagator at $p^2=0$ is unity. This prescription will ensure that the low-energy theorems for the scattering amplitudes are satisfied.

The lagrangian (4) is *not* the most general one with global $SU(2)_L \times SU(2)_R$ chiral symmetry; we have omitted a possible derivative coupling of the form $\kappa_L \bar{\psi}_L(\Sigma i\not\partial\Sigma^\dagger)\psi_L + \kappa_R \bar{\psi}_R(\Sigma^\dagger i\not\partial\Sigma)\psi_R$. (If parity is conserved, then $\kappa_L = \kappa_R$.) We also have not included any four-derivative terms involving Σ. To leading order in $1/N$, no such terms are needed to absorb divergences; the counterterm (5) suffices to cancel the divergences of the fermion loops in the WW→WW amplitudes.

The WW scattering amplitudes are given by

$$\begin{aligned}
&\mathcal{M}(Z^0Z^0 \to W^-W^+) = A(s,t,u)\,,\\
&\mathcal{M}(W^-W^+ \to Z^0Z^0) = A(s,t,u)\,,\\
&\mathcal{M}(W^-W^+ \to W^-W^+) = A(s,t,u) + A(t,s,u)\,,\\
&\mathcal{M}(Z^0Z^0 \to Z^0Z^0) = A(s,t,u) + A(t,s,u) + A(u,t,s)\,,\\
&\mathcal{M}(W^\pm Z^0 \to W^\pm Z^0) = A(t,s,u)\,,\\
&\mathcal{M}(W^\pm W^\pm \to W^\pm W^\pm) = A(t,s,u) + A(u,t,s)\,.
\end{aligned} \tag{6}$$

Including only diagrams which contribute to leading order in $1/N$, we find

$$A(s,t,u) = \frac{1}{N}\left(\frac{s}{v^2} - \frac{m^2}{4\pi^2 v^4} sF_2(2) - \frac{m^4}{4\pi^2 v^4}[F_4(s,t) + F_4(s,u) - F_4(t,u)]\right), \tag{7}$$

where

$$F_2(s) + \int_0^1 dx \ln\left(1 - \frac{s}{m^2}x(1-x) - i\epsilon\right),$$

$$F_4(s,t) = \int_0^1 dx \left(x^2 - x + \frac{m^2(s+t)}{st}\right)^{-1} \times\left[\ln\left(1 - \frac{s}{m^2}x(1-x) - i\epsilon\right) + \ln\left(1 - \frac{t}{m^2}x(1-x) - i\epsilon\right)\right]. \tag{8}$$

The integral $F_2(s)$ is given by

$$F_2(s<0) = -2 + \sqrt{1 - \frac{4m^2}{s}} \ln\left(\frac{\sqrt{4m^2-s} + \sqrt{-s}}{\sqrt{4m^2-s} - \sqrt{-s}}\right),$$

$$F_2(0<s<4m^2) = -2 + 2\sqrt{-1 + \frac{4m^2}{s}} \arctan\sqrt{\frac{s}{4m^2-s}},$$

$$F_2(s>4m^2) = -2 + \sqrt{1 - \frac{4m^2}{s}}\left[\ln\left(\frac{\sqrt{s} + \sqrt{s-4m^2}}{\sqrt{s} - \sqrt{s-4m^2}}\right) - i\pi\right], \tag{9}$$

and the integral $F_4(s,t)$ can be written [7] in terms of Spence functions as

$$\begin{aligned}
F_4(s,t) &= 2\left(1 - \frac{4m^2(s+t)}{st}\right)^{-1/2}\left[\mathrm{Sp}\left(\frac{x_+}{x_+ - y_+(s)} + i\sigma_s\epsilon\right)\right.\\
&+ \mathrm{Sp}\left(\frac{x_+}{x_+ - y_-(s)} - i\sigma_s\epsilon\right) - \mathrm{Sp}\left(\frac{-x_-}{x_+ - y_+(s)} + i\sigma_s\epsilon\right)\\
&- \mathrm{Sp}\left(\frac{-x_-}{x_+ - y_-(s)} - i\sigma_s\epsilon\right) + \mathrm{Sp}\left(\frac{x_+}{x_+ - y_+(t)} + i\sigma_t\epsilon\right)\\
&+ \mathrm{Sp}\left(\frac{x_+}{x_+ - y_-(t)} - i\sigma_t\epsilon\right) - \mathrm{Sp}\left(\frac{-x_-}{x_+ - y_+(t)} + i\sigma_t\epsilon\right)\\
&\left.- \mathrm{Sp}\left(\frac{-x_-}{x_+ - y_-(t)} - i\sigma_t\epsilon\right) + 2\pi i\Theta(st)\ln\left(\frac{x_+}{-x_-}\right)\right],
\end{aligned} \tag{10}$$

where σ_s denotes the sign of s, and

$$x_\pm = \tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} - \frac{m^2(s+t)}{st}},$$

$$y_\pm(s) = \tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} - \frac{m^2}{s}}. \tag{11}$$

Now we fix N to a finite value, the number of fermion doublets in the model. Then we set the scale v equal to $f/\sqrt{N}$, where $f=250$ GeV characterizes the scale of symmetry breaking, to obtain

$$A(s,t,u) = \frac{s}{f^2} - \frac{Nm^2}{4\pi^2 f^4} sF_2(s) - \frac{Nm^4}{4\pi^2 f^4}[F_4(s,t) + F_4(s,u) - F_4(t,u)]\,. \tag{12}$$

In the limit $s \ll m^2$, $A(s, t, u)$ approaches s/f^2, in accord with low-energy theorems.

Our model depends on only two parameters: the number of fermion doublets N and the fermion mass m. Experimental bounds require N to be $\leqslant 15$. This constraint derives from the contribution of additional heavy degenerate fermions to the shift in M_W [8].

We consider a model with $m = 250$ GeV and $N = 15$, corresponding to 5 additional generations of degenerate quark doublets, due to the color degeneracy. For these parameters, the partial waves obey unitarity ($|a_I^J| \leqslant 1$) for the energy region which we will consider [#1]. In table 1, we show the event rates and angular distributions of the W's produced in various modes at the SSC (with integrated luminosity 10^4 pb^{-1}) for this model, comparing them with results [#2] form the "low-energy theorem model," with lagrangian (2). We do not include W pairs produced by either quark or gluon fusion, restricting our consideration to WW scattering. The invariant mass of the W pair is required to be within 850 Gev and 1350 GeV. The branching ratio of the W boson decay has not been included. The transverse momentum of each decay product of the W's is required to be at least 20 GeV. No rapidity cut has been imposed on the final-state particles. The event rate is calculated using the effective-W approximation. The parton distribution function used is the leading order set, Fit SL, of Morfin and Tung [9]. The scale used in evaluating the parton distribution function in conjunction with the effective-W method is M_W.

From the results of table 1, we see that in the chiral lagrangian model with heavy fermion doublets, the rates for the W^+Z^0 and W^+W^+ modes are greatly reduced relative to the low-energy theorem model. In fact, the W^+W^+ event rate at $M_{WW} = 1.5$ TeV is down by about a factor of 10 from the prediction of the low-energy theorem model, and the W^+Z^0 mode is reduced by a similar factor. This is because the tree level amplitude is largely cancelled by the real part of the loop amplitude, and the imaginary part of the amplitude is zero for W^+W^+ and small for W^+Z^0. On the other hand, the heavy fermion model has a larger rate than the low-energy theorem model in both the W^-W^+ and Z^0Z^0 modes because of enhancement from the loop contribution to the scattering amplitudes. This increase, however, is less than a factor of 2 even at $M_{WW} = 1.5$ TeV. The event rate for the W^-W^+ and Z^0Z^0 modes in the TeV region is almost entirely due to the imaginary forward part of the amplitude (related by the optical theorem to the total cross section), the effect becoming stronger for higher M_{WW}.

In this letter, we have examined the effects of inelastic channels in the WW scattering process in one specific model. In this model, containing heavy fermion doublets, the rate of elastic WW scattering at energies above the threshold for fermion pair production differs significantly from low-energy theorem predictions. In particular, we found a large suppression of some elastic WW charge channels and an enhancement of others. A lesson to be drawn from this model is that all charge modes of the $WW \to WW$

Table 1
The event rates for W^+W^+, W^-W^+, Z^0Z^0, and W^+Z^0 production in one SSC year for the low-energy theorem model and for the chiral lagrangian model with heavy fermion doublets. The invariant mass of the W pair is required to satisfy $850 < M_{WW} < 1350$ GeV. The branching ratio of the W boson decay is not included. $0 < |\eta| < 1$ means that both W bosons have pseudo-rapidity between 0 and 1, etc.

		LET	HF
W^+W^+	$0<\|\eta\|<1$	94	22
	$0<\|\eta\|<2$	246	57
	$0<\|\eta\|<4$	343	80
	others	3	1
W^-W^+	$0<\|\eta\|<1$	199	333
	$0<\|\eta\|<2$	530	890
	$0<\|\eta\|<4$	741	1257
	others	6	11
Z^0Z^0	$0<\|\eta\|<1$	119	167
	$0<\|\eta\|<2$	308	444
	$0<\|\eta\|<4$	421	622
	others	3	6
W^+Z^0	$0<\|\eta\|<1$	88	22
	$0<\|\eta\|<2$	253	69
	$0<\|\eta\|<4$	373	110
	others	4	2

[#1] The partial waves in both our model (with these parameters) and the low-energy theorem model satisfy $|\mathrm{Re}\, a_I^J| \leqslant \frac{1}{2}$ up to about 1.2 TeV.

[#2] We use the scattering amplitudes given by Chanowitz and Gaillard, cited in ref. [1].

process need to be observed to be sure of detecting the symmetry-breaking sector.

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References

[1] M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. B 261 (1985) 379;
M.S. Chanowitz, Annu. Rev. Nucl. Part. Sci. 38 (1988) 323.
[2] M.S. Chanowitz and M. Golden, Phys. Rev. Lett. 61 (1988) 1053; 63 (1989) 466(E);
M.S. Berger and M.S. Chanowitz, preprint LBL-30476 (1991).
[3] R.S. Chivukula and M. Golden, Phys. Lett. B 267 (1991) 233.
[4] S.G. Naculich and C.-P. Yuan, preprint JHU-TIPAC-920017.
[5] J. Bagger, S. Dawson and G. Valencia, preprint BNL-45782; preprint JHU-TIPAC-920009.
[6] S.G. Naculich and C.-P. Yuan, in preparation.
[7] G. 't Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365.
[8] M. Veltman, Phys. Lett. B 91 (1980) 95;
W.J. Marciano and A. Sirlin, Phys. Rev. D 22 (1980) 2695;
S. Bertollini and A. Sirlin, Nucl. Phys. B 248 (1984) 589.
[9] J.G. Morfin and W.K. Tung, Z. Phys. C 52 (1991) 13.