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### Unusual decay modes of D0 and D+ mesons

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## ARTICLES

Unusual decay modes of  $D^0$  and  $D^+$  mesons

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CLEO has measured decay modes of the  $D^0$  and  $D^+$  into final states consisting of  $K^\pm$ 's,  $\pi^\pm$ 's,  $K^0$ 's and  $\bar{K}^0$ 's, using data taken with the CLEO detector at the Cornell Electron Storage Ring. We report new results on the decays of  $D^0$ 's into  $4\pi^\pm$ ,  $K^-K^+\pi^-\pi^+$ ,  $\bar{K}^0K^+K^-$ ,  $\bar{K}^0K^+\pi^-$ ,  $K^0K^-\pi^+$ ,  $3K_S^0$  and  $\bar{K}^0\phi$  together with some of their resonant substructure. We also present the first observation of the decay  $D^+ \rightarrow \bar{K}^0\bar{K}^0K^+$  and give limits on the doubly-Cabibbo-suppressed decays of the  $D^0$  into  $K^+\pi^-$  and  $K^+\pi^-\pi^+\pi^-$ .

## INTRODUCTION

Since the discovery of charmed mesons in 1976 [1] there have been many measurements of their decay properties. Measurements of the relative lifetimes and semileptonic branching fractions of the  $D^0$  and  $D^+$  immediately lead to the conclusion that the simple spectator model of charmed-meson decay is inadequate. More detailed measurements of exclusive decay modes of the  $D^0$  and  $D^+$  have helped estimate the contributions of  $W$ -

exchange diagrams and final-state interactions to their decays. The measurements of the  $D^0$  branching fractions have almost reached the point where all the decays are accounted for, but the measurements of the  $D^+$  are not so well advanced. This paper reports new measurements of the decay rates of several unusual decay modes of the  $D^0$  together with one of the  $D^+$ . These rates are measured relative to those of well-known decay modes. All the decay modes reported here involve final states with no photons; other decay modes involving photonic final

states have been reported elsewhere [2].

The data used for this study were taken by the CLEO detector operating at the Cornell Electron Storage Ring. The data sample used comprises  $466 \text{ pb}^{-1}$  in the  $\Upsilon$  energy range, including data taken at the  $\Upsilon(3S)$ ,  $\Upsilon(4S)$ , and  $\Upsilon(5S)$  resonances. The trajectories of charged particles produced in the collisions were measured using a set of three concentric drift chambers inside a 1-T magnetic field produced by a 1-m-radius solenoid. The innermost of these three was a three-layer "straw tube" vertex detector with a position resolution of  $70 \mu$  in the  $r$ - $\phi$  plane. The middle detector was a 10-layer high-resolution drift chamber with a position resolution of  $90 \mu$  in the  $r$ - $\phi$  plane. The main drift chamber comprised 51 layers each with a position resolution of  $110 \mu$ . Measurement of the along-the-beam coordinate of the charged particles was obtained by a combination of stereo layers in the main drift chamber together with cathode strips. The tracking system had a momentum resolution given by  $(dp/p)^2 = (0.23\%)^2 + (0.7\%)^2$ , where  $p$  is in  $\text{GeV}/c$ . The CLEO method of selecting hadronic events has been explained elsewhere [3]. Protons, kaons, and pions were identified by a measurement of the specific ionization ( $dE/dx$ ) of the particles in the main drift chamber. Individual charged particles were assigned the kaon identity if their measured  $dE/dx$  was within  $2\sigma$  of that expected for a kaon of that momentum. They were assigned a  $\pi$  identity if their  $dE/dx$  was within  $3\sigma$  of that expected for a  $\pi$ . All particles that had measured  $dE/dx$  greater than  $2\sigma$  away from that expected for a kaon or proton, or did not have good  $dE/dx$  information, were also assigned the pion identity. Particles could be assigned both  $K$  and  $\pi$  identities.

Neutral  $K$ 's were identified by the decay  $K_S^0 \rightarrow \pi^+ \pi^-$ . These were detected as a  $\pi^+ \pi^-$  pair with invariant mass consistent with the  $K_S^0$  and a decay point separated from the main event vertex. The exact definition of a  $K_S^0$  candidate depended upon the mode under study, as the detection of some modes were background limited and others statistics limited. All the  $D^0$  modes (except that into  $K_S^0 K_S^0 K_S^0$ ) were detected using the observation of the decay  $D^{*+} \rightarrow D^0 \pi^+$ . A point of the mass difference  $m(D^{*+}) - m(D^0)$  yields a peak at a value of  $145.5 \text{ MeV}/c^2$ , with a width of only  $1.6 \text{ MeV}/c^2$  full width at half maximum (FWHM). The mass difference plots presented here (with the exception of those that are part of the search for doubly-Cabibbo-suppressed modes) were obtained in the following manner. A  $D^0$  signal band was defined by those combinations within a certain mass band around the known  $D^0$  mass. The size of this band is dependent on the mode. Sideband regions were defined above and below this mass band, and the mass difference plot was obtained for signal minus sideband regions of  $D^0$  mass. This yields a "background-subtracted" mass plot. The technique of subtracting  $D^0$  sidebands can also then be used to search for resonant substructure to the modes. One advantage of this procedure compared with simply fitting the mass difference plot is that it avoids the necessity of fitting the complicated shape of the combinatorial background in the latter. It also avoids the false peaks that can arise, for instance, from misidentification of both

a  $K^-$  as a  $\pi^-$  together with misidentification of a  $\pi^+$  as a  $K^+$ , misidentification of a  $\pi$  as a  $K$  together with missing a  $\pi^0$ , etc.

For most modes, the  $D^0$  candidates are required to have a large  $x_p$ , where  $x_p$  is defined to be the ratio of the particle's momentum to its maximum possible momentum, in order to improve the signal-to-background ratio. For a few modes that are not background limited the re-

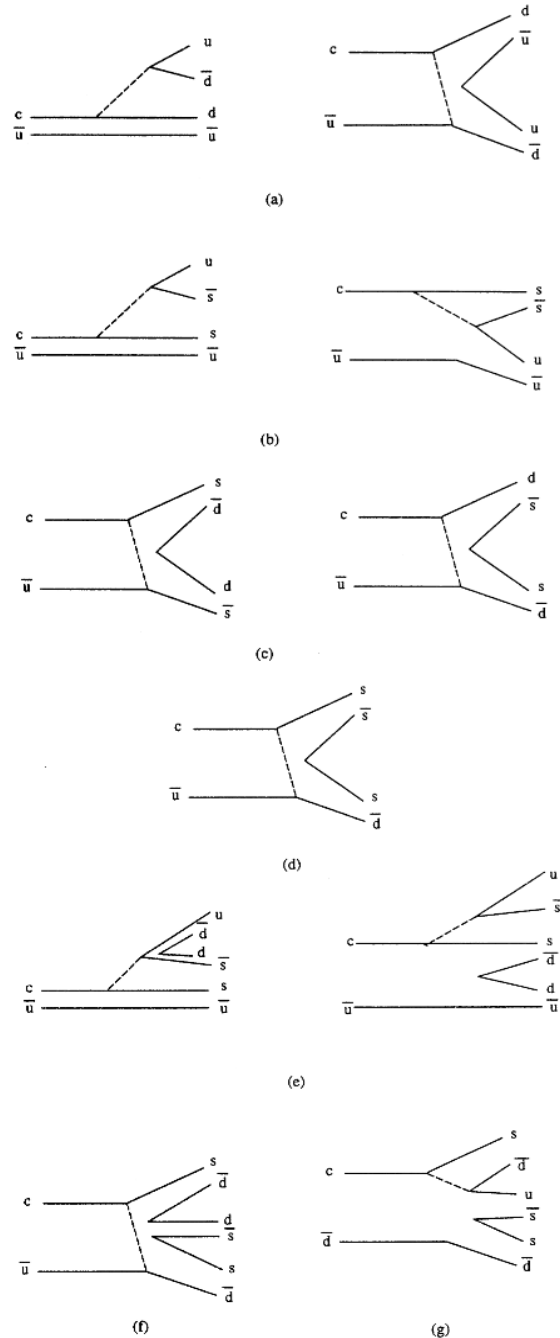


FIG. 1. Typical Feynman diagrams for decays that lead to final states of (a)  $\pi^+ \pi^- \pi^+ \pi^-$ , (b)  $K^+ K^- \pi^+ \pi^-$ , (c)  $\bar{K}^{*0} K^{*0}$ , (d)  $\phi \bar{K}^0$ , (e)  $\bar{K}^0 K \pi$ , (f)  $K_S^0 K_S^0 K_S^0$ , (g)  $\bar{K}^0 \bar{K}^0 K^+$ .

TABLE I. Summary of  $D$  relative branching ratios. "Set 1" refers to a  $D^0$  mass of 1.84–1.89 GeV/ $c^2$ , a low sideband of 1.805–1.830 GeV/ $c^2$ , and a high sideband of 1.900–1.925 GeV/ $c^2$ . "Set 2" refers to a  $D^0$  mass of 1.845–1.885 GeV/ $c^2$ , a low sideband of 1.795–1.835 GeV/ $c^2$ , and a high sideband of 1.895–1.935 GeV/ $c^2$ .

Decay mode	$x_p$ cut	Mass cuts	Yield	Relative efficiency	Branching ratio
$D^0$ decays					
$\frac{\pi^+\pi^-\pi^+\pi^-}{K^-\pi^+\pi^-\pi^+}$	0.5	set 1 set 2	$\frac{345 \pm 48}{2933 \pm 67}$	1.15	$0.102 \pm 0.013$
$\frac{K^+K^-\pi^+\pi^-}{K^-\pi^+\pi^-\pi^+}$	0.4	set 2 set 2	$\frac{89 \pm 29}{3628 \pm 80}$	0.79	$0.0314 \pm 0.010$
$\frac{\phi\rho}{K^-\pi^+\pi^-\pi^+}$	0.4	set 2 set 2	$\frac{34.5 \pm 8.6}{3628 \pm 80}$	0.39	$0.024 \pm 0.006$
$\frac{\bar{K}^{*0}K^{*0}}{K^-i^+\pi^-\pi^+}$		set 2 set 2	$\frac{11 \pm 7}{3628 \pm 80}$	0.19	$< 0.19$
$\frac{K^+\pi^-}{K^-\pi^+}$	0.6		$\frac{< 4.3}{420 \pm 21}$	1.00	$< 0.011$
$\frac{K^+\pi^-\pi^+\pi^-}{K^-\pi^+\pi^-\pi^+}$	0.6		$\frac{< 6.50}{393 \pm 20}$	1.0	$< 0.18$
$\frac{\phi\bar{K}^0}{\bar{K}^0\pi^+\pi^-}$	0.0	set 2 set 1	$\frac{63 \pm 9}{1055 \pm 50}$	0.36	$0.163 \pm 0.023$
$\frac{\bar{K}^0K^+K^-}{\bar{K}^0\pi^+\pi^-}$	0.0	set 2 set 1	$\frac{136 \pm 13}{616 \pm 29}$	0.75	$0.170 \pm 0.022$
$\frac{K^0K^-\pi^+}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{61 \pm 10}{616 \pm 29}$	0.90	$0.108 \pm 0.019$
$\frac{\bar{K}^0K^+\pi^-}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{55 \pm 11}{616 \pm 29}$	0.90	$0.098 \pm 0.020$
$\frac{K^{*-}K^+}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{23 \pm 6}{616 \pm 29}$	0.62	$0.064 \pm 0.018$
$\frac{K^{*-}K^+}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{12 \pm 7}{616 \pm 29}$	0.62	$0.034 \pm 0.019$
$\frac{K^{*0}\bar{K}^0}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{0 \pm 5}{616 \pm 29}$	0.62	$< 0.015$
$\frac{K^0\bar{K}^{*0}}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{6 \pm 5}{616 \pm 29}$	0.62	$< 0.029$
$\frac{\bar{K}^0\pi^+\pi^-\pi^+\pi^-}{\bar{K}^0\pi^+\pi^-}$	0.5	set 2 set 1	$\frac{56 \pm 11}{616 \pm 29}$	0.61	$0.149 \pm 0.026$
$\frac{K_S^0K_S^0K_S^0}{\bar{K}^0\pi^+\pi^-}$	0.5		$\frac{22 \pm 5}{3298 \pm 114}$	0.33	$0.016 \pm 0.005$
$D^+$ decays					
$\frac{\bar{K}^0\bar{K}^0K^+}{K^-\pi^+\pi^+}$	0.5		$\frac{70 \pm 12}{4639 \pm 213}$	0.044	$0.34 \pm 0.07$

quirement is decreased. Requirements on  $x_p$  are listed in Table I for all modes investigated. For  $x_p > 0.5$  all the charmed particles found are produced directly; i.e., they are not the decay products of  $B$  mesons. However, as

this paper sets out only to measure the relative branching fractions of  $D$  mesons, their production mechanism is not relevant. Now we describe the particles of the detection of each  $D$  decay mode studied.

### THE DECAY $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

This decay is Cabibbo suppressed; sample Feynman diagrams leading to this final state are shown in Fig. 1(a). For this decay mode the  $D^0$  mass band was defined to be  $1.84\text{--}1.89\text{ GeV}/c^2$ , the sidebands used for background subtraction were  $1.805\text{--}1.83\text{ GeV}/c^2$  and  $1.90\text{--}1.925\text{ GeV}/c^2$ , and the combinations were required to have  $x_p > 0.5$ . Decays of the  $D^0$  into  $K^- \pi^+ \pi^- \pi^+$  when mistakenly reconstructed as  $\pi^+ \pi^- \pi^+ \pi^-$  populate regions below the low-mass sideband. The background-subtracted mass-difference plot is shown in Fig. 2(a). The fit yields  $429 \pm 45$   $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ 's. Those  $\pi$ 's that are part of reconstructed  $K_S^0$ 's are not used in this analysis; however, there remain some  $D^0$  decays into  $K_S^0 \pi^+ \pi^-$  where the  $K_S^0$  is not identified as a separated vertex. Defining  $D^0$  candidates as those with a  $4\pi$  mass within

the signal region and a mass difference of  $0.1435\text{--}0.1475\text{ MeV}/c^2$ , a plot of the invariant mass of all  $\pi^+ \pi^-$  combinations in these candidates is shown in Fig. 2(b). There is a clear  $K_S^0$  peak, with a fit area of 84 events. This number is consistent with the expected background due to  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ . These are thus subtracted from the  $4\pi$  yield quoted above to get a final yield of  $345 \pm 50$ . This decay mode is then compared with the Cabibbo-allowed decay mode  $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$  analyzed in a similar way. The relative efficiency of the modes is determined by Monte Carlo simulation to be 1.15. This number is greater than unity because of the lower efficiency of assigning a kaon the correct identity compared to the analogous pion efficiency as well as the greater likelihood of a kaon decaying in flight compared with a pion. The final result is that the ratio of decay rates

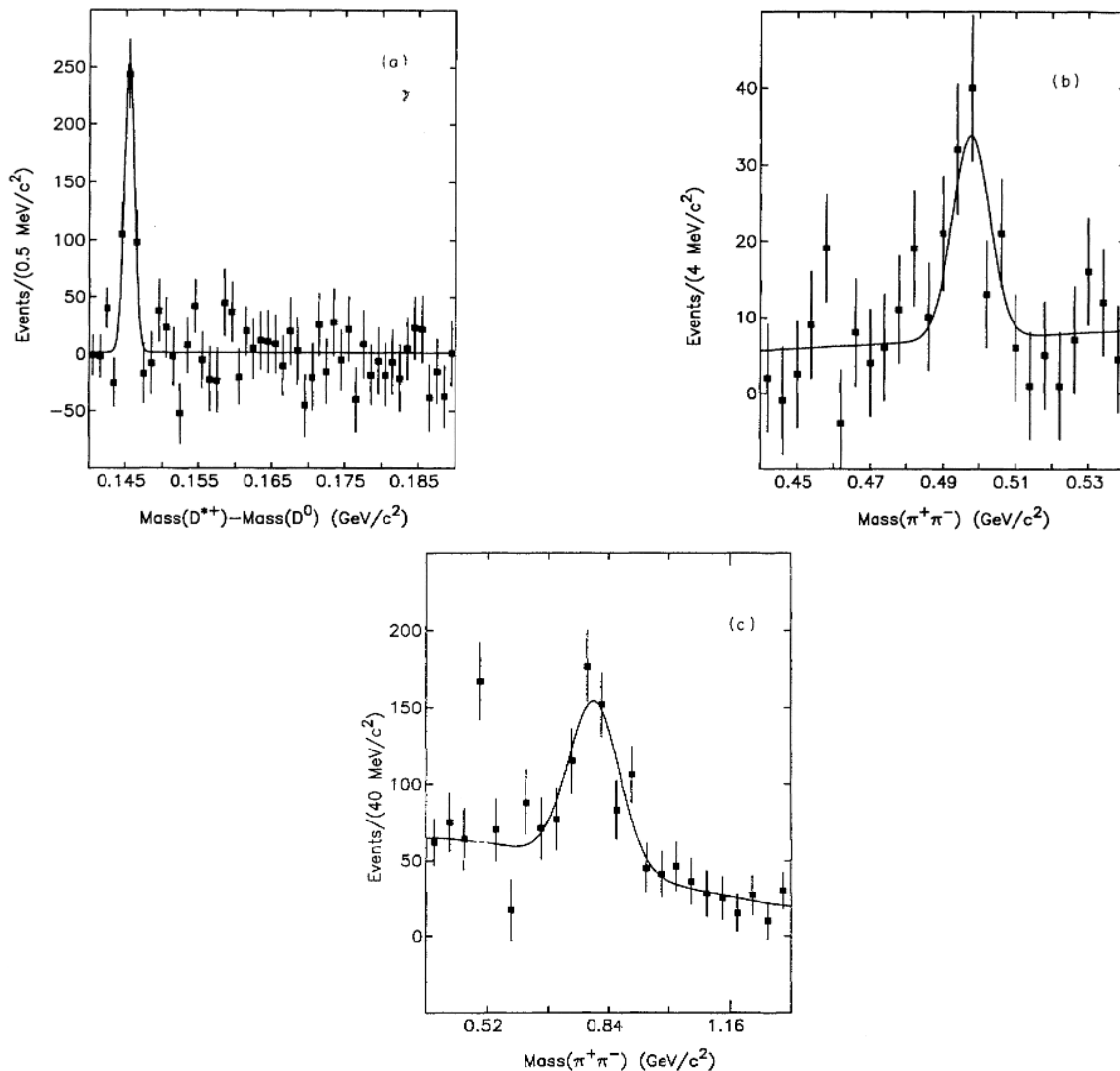


FIG. 2. (a) Background-subtracted  $D^{*+}\text{--}D^0$  mass-difference plot, where  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ . (b) Background-subtracted  $\pi^+ \pi^-$  mass spectrum for tracks included in the  $D^{*+}$  peak in (a); the  $K_S^0$  region is shown. (c) Background-subtracted  $\pi^+ \pi^-$  mass spectrum for tracks included in the  $D^{*+}$  peak in (a); the  $\rho^0$  region is shown. The fit excludes the  $K_S^0$  region.

$$B(D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-) / B(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+) = 0.102 \pm 0.013$$

(see Table I). This number is considerably larger than the 1990 world average [4], in particular it appears to be inconsistent with the number obtained by the Amsterdam-Bristol-CERN-Cracow-Rutherford (ACCMOR) Collaboration [5]. However, it is very similar to the recent value obtained by the E-691 Collaboration [6]. An invariant-mass plot of all  $\pi^+ \pi^-$  pairs within the  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  candidates shows a clear  $\rho^0$  peak, with an area of  $447 \pm 72$  events [Fig. 2(c)]. By also fitting the  $\rho^0$  peak in the  $\pi^+ \pi^-$  distribution in  $K_S^0 \pi^+ \pi^-$  events where the  $K_S^0$  has been identified, we can estimate that  $17 \pm 4$  of the 447 events come from this source. Thus there are  $1.25 \pm 0.25 \pm 0.25$   $\rho$ 's per  $\pi^+ \pi^- \pi^+ \pi^-$  decay, where the systematic error quoted is an estimate of the uncertainty

in the fitting procedure. There can be contributions to this mode from  $D^0 \rightarrow \rho^0 \rho^0$ ,  $D^0 \rightarrow a_1^- \pi^+$ , and  $D^0 \rightarrow a_1^+ \pi^-$ , where  $a_1 \rightarrow \rho \pi$  and  $\rho^0 \rightarrow \pi^+ \pi^-$ , but the data are not sufficient to disentangle this resonant substructure. Nonetheless, our result implies that the  $\pi^+ \pi^- \pi^+ \pi^-$  are not produced through pure phase space.

#### THE DECAY $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$

This decay mode is also Cabibbo suppressed; typical diagrams are shown in Figs. 1(b) and 1(c). The analysis proceeds in a similar way to that of  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ . There is a more restrictive requirement on the  $D^0$  mass (see Table I) because of the narrower signal in this mode, and the  $x_p$  cut is lowered to 0.4 to obtain more statistical precision. There is a further requirement on the  $dE/dx$  information of the two tracks designated  $K$ 's. The sum of

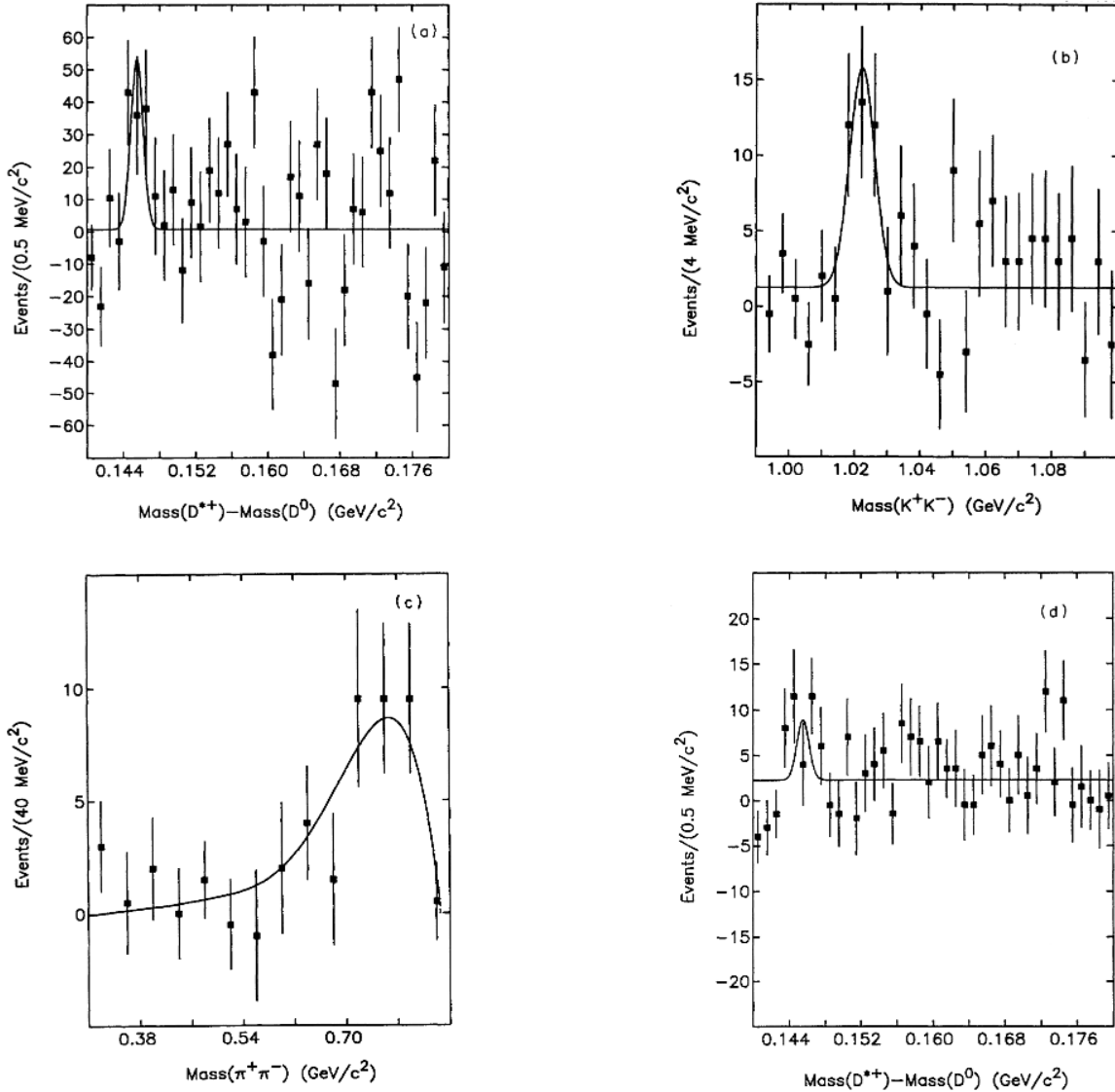


FIG. 3. (a) Background-subtracted  $D^{*+} - D^0$  mass-difference plot, where  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ . (b) Background-subtracted  $K^+ K^-$  mass-difference plot for tracks included in the  $D^{*+}$  peak in (a). (c) Background-subtracted  $\pi^+ \pi^-$  mass plot for tracks included in the  $D^{*+}$  (a). The fit is explained in the text. (d) Background-subtracted  $D^{*+} - D^0$  mass-difference plot, where  $D^0 \rightarrow \bar{K}^{*0} K^{*0}$ .

the squares of the deviations of the measured  $dE/dx$  from the expected  $dE/dx$  is required to be less than 4 for these two tracks in order to reduce pion background. Combinations that have a  $\pi^+\pi^-$  mass consistent with the  $K_S^0$  mass are excluded from this analysis; this cut results in only a small lowering of the efficiency. The background-subtracted mass-difference plot is shown in Fig. 3(a) and the fit gives a total yield in this mode of  $97 \pm 27$   $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  events. This yields a ratio of branching fractions  $B(K^+ K^- \pi^+ \pi^-) / B(K^- \pi^+ \pi^- \pi^+)$  of  $0.031 \pm 0.010$  (see Table I).

Taking those combinations with a mass difference of  $0.1435 - 0.1475$   $\text{GeV}/c^2$ , the background-subtracted  $K^+ K^-$  invariant mass is plotted for the candidates. There is a clear  $\phi$  signal [Fig. 3(b)]. Furthermore, Fig. 3(c) shows the invariant mass of the  $\pi^+ \pi^-$  pair for those decays that have a  $K^+ K^-$  mass consistent with a  $\phi$ . The mass spectrum is consistent with being entirely due to  $\rho$ 's and is fit to a  $\rho$  shape with phase-space suppression taken into account. It is clear from this plot that any contribution to this mode from nonresonant  $\phi \pi^+ \pi^-$  is small as Monte Carlo modeling predicts that the  $\pi^+ \pi^-$  invariant-mass plot for such events peaks at around  $0.5$   $\text{GeV}/c^2$ . This fit yields a  $\phi\rho$  signal of  $34.5 \pm 8.6$  events, where the statistical uncertainty includes the possibility of  $\phi \pi^+ \pi^-$  nonresonant decays. This corresponds to a branching fraction of  $0.19 \pm 0.05\%$  into this mode. A signal in  $D^0 \rightarrow \phi \pi^+ \pi^-$  has previously been reported by the ACCMOR Collaboration [7], and E-691 [6] have recently published an upper limit of  $0.15\%$  in this mode. Neither of the experiments has the statistics necessary to separate  $\phi\rho$  from  $\phi \pi^+ \pi^-$  (nonresonant). Another possible substructure in the  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  events is  $D^0 \rightarrow K^{*0} \bar{K}^{*0}$ . This mode is particularly interesting as not only can it not occur through simple spectator decay diagrams, but furthermore the  $W$ -exchange diagram [Fig. 1(c)] is Glashow-Iliopoulos-Maiani (GIM) suppressed. Thus evidence of this mode would be a strong indication of the importance of final-state interactions. This is explained in more detail in our previous Letter detailing the observation of  $D^0 \rightarrow K^0 \bar{K}^0$  [8]. To search for this mode, we define  $D^{*0}$  ( $\bar{K}^{*0}$ )'s as those  $K^- \pi^+$  ( $K^+ \pi^-$ ) combinations with an invariant mass within  $40$   $\text{MeV}/c^2$  of the  $K^{*0}$  mass. A small number of these combinations are also in the  $\phi\rho$  sample defined above. The background-subtracted  $D^{*+} - D^0$  mass-difference plot is shown in Fig. 3(d). Although there appears to be some enhancement in the  $D^{*+}$  region, the fit does not yield a statistically significant signal, and we prefer to quote an upper limit for this mode. We have not made any subtraction for the feedthrough into  $K^{*0} \bar{K}^{*0}$  from  $\phi\rho$ ,  $K^{*0} K^-$ ,  $\bar{K}^{*0} K^+$ , or  $K^+ K^- \pi^+ \pi^-$  (nonresonant). We obtain an upper limit of  $0.26\%$ , which includes the added systematic uncertainty due to the modeling of the phase-space suppression of the resonances. This limit is lower than the signal claimed by the E-691 experiment [6].

#### THE DECAYS $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ AND $D^0 \rightarrow K^+ \pi^-$

Weak decays present an opportunity of observing mixing between particle and antiparticle. Observation of this effect in both the strange and bottom sector has also been

very useful in considerations of the origin of  $CP$  violation. In general, the likelihood is proportional to the quotient  $\Delta_{+-}/\Gamma$ , where  $\Delta_{+-}$  is the mass difference not between the mass eigenstates  $D^0$  and  $\bar{D}^0$ , but rather the  $CP$  eigenstates  $D_L$  and  $D_S$ . Unlike the bottom system, the charm quark can decay within its own generation; correspondingly, the mixing rate is expected to be much smaller than the Cabibbo-allowed simple  $c \rightarrow s$  transition.

Definitive evidence for either mixing or doubly-Cabibbo-suppressed  $D^0$  decays (DCSD's) has so far not been found. The standard technique for observation of such decays is searching for  $D^{*}$  decays where the soft pion has the same charge as the  $K$  arising from the spectator decay of the neutral  $D$ . Without measuring the time evolution of the final state it is impossible to differentiate between DCSD's and  $D^0 \bar{D}^0$  mixing.

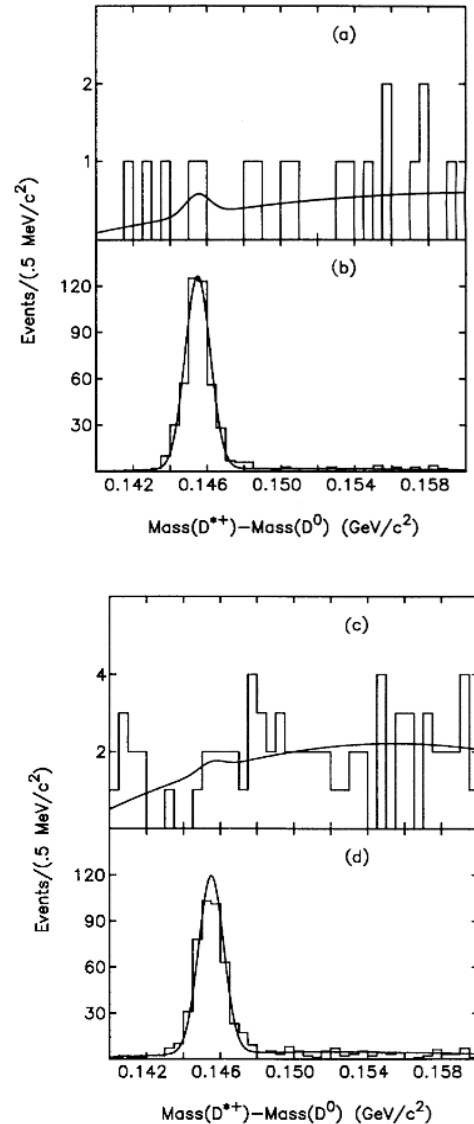


FIG. 4. (a)  $D^{*+} - D^0$  mass-difference plots for (a)  $D^0 \rightarrow K^+ \pi^-$ , (b)  $D^0 \rightarrow K^- \pi^+$ , (c)  $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ , and (d)  $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ .



In order to make the search for these decays as sensitive as possible, an analysis technique was employed which differs from that used for the other decay modes reported in this paper. Rather than use a sideband subtraction technique to account for the background, very stringent cuts were applied with a view to eliminating all background.  $K^+\pi^-$  candidates were tagged by the decay  $D^{*+} \rightarrow D^0, D^0 \rightarrow K^+\pi^-$ , with the  $K^+\pi^-$  mass required to be within 15  $\text{MeV}/c^2$  of the  $D^0$  mass. A function corresponding to the probability of the  $dE/dx$  of the two tracks being due to a  $K^+$  and a  $\pi^-$  was calculated and this probability was required to be greater than 0.4. In order to further discriminate against  $K^-\pi^+$  combinations, the invariant mass of the  $K^+\pi^-$  candidates, when interpreted as  $K^-\pi^+$ 's, were required to be more than 60  $\text{MeV}/c^2$  away from the  $D^0$  mass. Lastly a very restrictive  $x_p$  requirement of 0.6 was made. Figure 4(a) shows the resultant mass-difference plot. The fit shown is the best fit to a  $D^0$  signal with fixed mass and width, but no statistically significant signal is observed. The 90% confidence level limit to the number of produced combinations is 4.3. In order to provide a suitable normalization, the analogous cuts were applied to obtain a signal in  $D^0 \rightarrow K^-\pi^+$  [Fig. 4(b)], where a signal of  $420 \pm 21$  is observed. Assuming the efficiencies of  $K^-\pi^+$  and  $K^+\pi^-$  are identical when analyzed in this manner, an upper limit on the ratio of branching fractions  $B(D^0 \rightarrow KK^+\pi^-)/B(D^0 \rightarrow KK^-\pi^+)$  of 0.011 is obtained. This is an improvement on the present world average [4] for the limit on the doubly-Cabibbo-suppressed mode  $B(D^0 \rightarrow K^+\pi^-)$ ; E-691 have obtained a more stringent limit for this final state occurring through  $D^0\bar{D}^0$  mixing [9].

For the case of  $K^+\pi^-\pi^+\pi^-$  a similar analysis technique was used, but with slightly more stringent mass requirements since the resolution of the  $D^0$  mass is better. The  $K^+\pi^-\pi^+\pi^-$  combination was required to be within 10  $\text{MeV}/c^2$  of the  $D^0$  mass; with both possible  $K^-\pi^+\pi^-\pi^+$  interpretations required to be greater than 40  $\text{MeV}/c^2$  from the  $D^0$  mass. In this analysis the probability function was constructed with the  $dE/dx$  information from all four tracks, with the requirement on this probability at 0.4 as before. The mass-difference plot for this mode is shown in Fig. 4(c), together with that for the decay  $K^-\pi^+\pi^-\pi^+$  analyzed in the same way [Fig. 4(d)]. Once again, a 90% confidence-level upper limit is obtained:

$$B(D^0 \rightarrow K^+\pi^-\pi^+\pi^-)/B(D^0 \rightarrow K^-\pi^+\pi^-\pi^+) < 0.018.$$

The naive expectation for the ratio of the DCSD mode to the Cabibbo-allowed analogue is that it should be  $\tan^4\theta_C \approx 0.002$ . The data are not sufficient to test this expectation. This limit is at a similar level to the present world average [4], dominated by the E-691 result [9].

#### THE DECAY $D^0 \rightarrow \bar{K}^0 K^+ K^-$

The Cabibbo-allowed decay  $D^0 \rightarrow \bar{K}^0 K^+ K^-$  has been the object of considerable interest for some years. In particular, the first observations [10] of the decay chain  $D^0 \rightarrow \phi \bar{K}^0$  were interpreted as firm evidence of  $W$ -

exchange diagrams [Fig. 1(d)] in  $D$  decays, as a simple spectator model predicts a zero amplitude. More recent theoretical considerations have led to the conclusion that final-state rescattering makes a large contribution to this decay [11], but this assertion remains controversial.

The analysis proceeds in a similar manner to the modes above. In this case, as the signal-to-background ratio is so high, no  $x_p$  cut is applied. The  $\bar{K}^0$ 's are detected through their decay  $K_S^0 \rightarrow \pi^+\pi^-$ . The two  $\pi$ 's that make up the  $K_S^0$  are required to have a point of intersection separated by at least 5 mm from the event vertex and to have a mass within 12  $\text{MeV}/c^2$  of the  $K_S^0$ . The background-subtracted mass-difference plot for this mode is shown in Fig. 5(a); it shows a signal of  $136 \pm 13$  events. The branching fraction for this mode is compared with

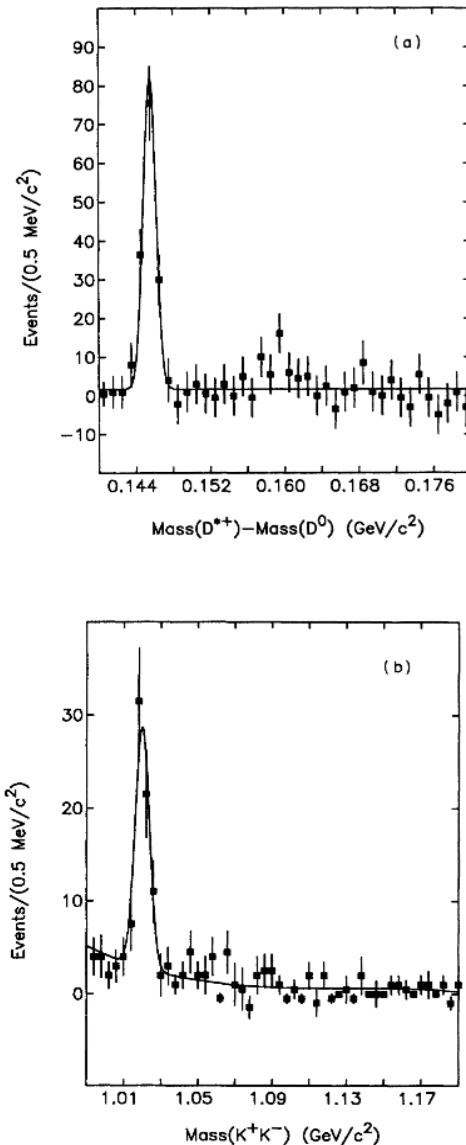


FIG. 5. (a) Background-subtracted  $D^{*+}$ - $D^0$  mass-difference plot where  $D^0 \rightarrow \bar{K}^0 K^+ K^-$ . (b) Background-subtracted  $K^+ K^-$  invariant mass for tracks included in the  $D^{*+}$  peak of (a).

TABLE II. Summary of  $D$  absolute branching fractions.

Decay mode	Branching fraction (%)	Previous measurement (%)	Theory (%)
$D^0$ decays			
$\pi^+\pi^-\pi^+\pi^-$	$0.80\pm 0.14$	$0.35^{+0.14}_{-0.012}$ [4] $0.87\pm 0.16\pm 0.13$ [6]	
$K^+K^-\pi^+\pi^-$	$0.24\pm 0.08$	$0.32\pm 0.08$ [4] $0.26^{+0.07}_{-0.06}\pm 0.05$ [6]	
$\phi\rho$	$0.19\pm 0.05$	$0.30\pm 0.10$ [4] $< 0.15$ [6]	$0.61$ [14]
$K^+\pi^-$	$< 0.04$	$< 0.06$ [4]	
$K^+\pi^-\pi^+\pi^-$	$< 0.14$	$< 0.14$ [9]	
$K^0K^-\pi^+$ (including $K^{*+}$ )	$0.58\pm 0.10$	$0.8\pm 0.4$ [4] (with c.c.) $0.84\pm 0.17$ [12] $0.69^{+0.27}_{-0.23}\pm 0.14$ [6]	
$\bar{K}^0K^+\pi^-$ (including $K^{*-}$ )	$0.52\pm 0.11$	$0.80\pm 0.40$ [4] (with c.c.) $0.42^{+0.23}_{-0.20}\pm 0.09$ [6]	
$K^{*+}K^-$	$0.34\pm 0.10$	$0.8\pm 0.4$ [4] (with c.c.) $0.5\pm 0.2\pm 0.1$ [12] $0.69^{+0.32}_{-0.26}\pm 0.14$ [6]	$0.55$ [14] $0.66$ [15]
$K^{*-}K^+$	$< 0.30$	$0.8\pm 0.4$ [4] (with c.c.) $< 0.17$ [6]	$0.20$ [14] $0.40$ [15]
$K^{*0}\bar{K}^0$	$< 0.08$	$< 0.5$ [4] (with c.c.) $< 0.22$ [6]	
$\bar{K}^{*0}K^0$	$< 0.15$	$< 0.5$ [4] (with c.c.) $< 0.13$ [6] (with c.c.)	
$\phi\bar{K}^0$	$0.86\pm 0.14$	$0.80\pm 0.16$ [4]	$0.0$ [14] $1.3$ [15]
$\bar{K}^0K^+K^-$ (including $\phi\bar{K}^0$ )	$0.96\pm 0.15$	$1.16\pm 0.21$ [4]	
$\bar{K}^0\pi^+\pi^-\pi^+\pi^-$	$0.78\pm 0.16$	$0.7\pm 0.4$ [4]	
$K_S^0K_S^0K_S^0$	$0.11\pm 0.03$	$0.09\pm 0.04$ [12]	
$D^+$ decays			
$\bar{K}^0\bar{K}^0K^+$	$2.6\pm 0.6$		

that of  $K^0\pi^+\pi^-$  using the same criteria for identifying  $K_S^0$ 's and the results presented in Table I.

Taking those combinations in Fig. 5(a) with a mass difference between 0.1435 and 0.1475 MeV/ $c^2$ , the  $K^+K^-$  invariant-mass distribution was calculated [Fig. 5(b)]. This shows a large enhancement, of  $63\pm 9$  events, in the  $\phi$  region. Thus  $0.46\pm 0.05$  of the  $\bar{K}^0K^+K^-$  events are due to  $\bar{K}^0\phi$ . The fit shown in Fig. 5(b) is a Gaussian with a polynomial background. The shape of the background peaks at low mass, below the  $\phi$  and close to the

kinematic limit. This is indicative of a contribution from the  $a_0(980)$  which decays into  $K^+K^-$  with a central mass value below threshold, but with a long tail extending to higher masses. This contribution from the  $a_0$  could account for all of the non- $\phi$  component of the  $\bar{K}^0K^+K^-$ . The existence of the  $a_0$  does not appreciably affect the measured yield of  $\bar{K}^0\phi$  events. The branching fraction obtained for this mode (Table II) is in good agreement with, and of comparable accuracy to, the present world average.

### THE DECAYS $D^0 \rightarrow K^0 K^- \pi^+$ AND $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$

These two modes have different decay diagrams [Fig. 1(e)], and so are not necessarily the same. They are separated by considering the charge correlation between the soft pion in the  $D^*$  decay and the pion in the  $D^0$  decay. The criteria for detecting  $K_S^0$ 's are sufficiently tight to effectively reduce the background from  $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$  to less than one event. Figure 6(a) shows the background-subtracted mass-difference plot for  $K_S^0 K^- \pi^+$  and the fit yields a total of  $61 \pm 12$  events. The similar figure for  $K_S^0 K^+ \pi^-$  [Fig. 6(b)] shows  $55 \pm 11$  events. It is particularly interesting to look at the sub-

structure of these modes, as two-body decays are much more amenable to theoretical calculation than three-body decays. In the simplest picture of  $D^0$  decays,  $K^{*+} K^-$  arises when the  $W^+$  hadronizes as a vector, and the spectator hadronizes as a pseudoscalar, whereas in  $K^{*+} K^+$  the situation is reversed. Figure 6(c) shows the  $K_S^0 \pi^+$  invariant mass for background-subtracted  $D^0 \rightarrow K_S^0 K^- \pi^+$  candidates. It shows a clear  $K^{*+}$  signal. On the other hand, the analogous figure [Fig. 6(d)] for  $K^{*+} K^+$  shows no clear  $K^{*+}$  peak. The results are tabulated in Table I. We also searched for evidence of  $K^{*0} \bar{K}^0$  and  $\bar{K}^{*0} K^0$  decays. Figure 6(e) shows the  $K^+ \pi^-$  invariant-mass plot in  $K_S^0 K^+ \pi^-$  events; it shows no signal at the  $K^{*0}$ . Figure

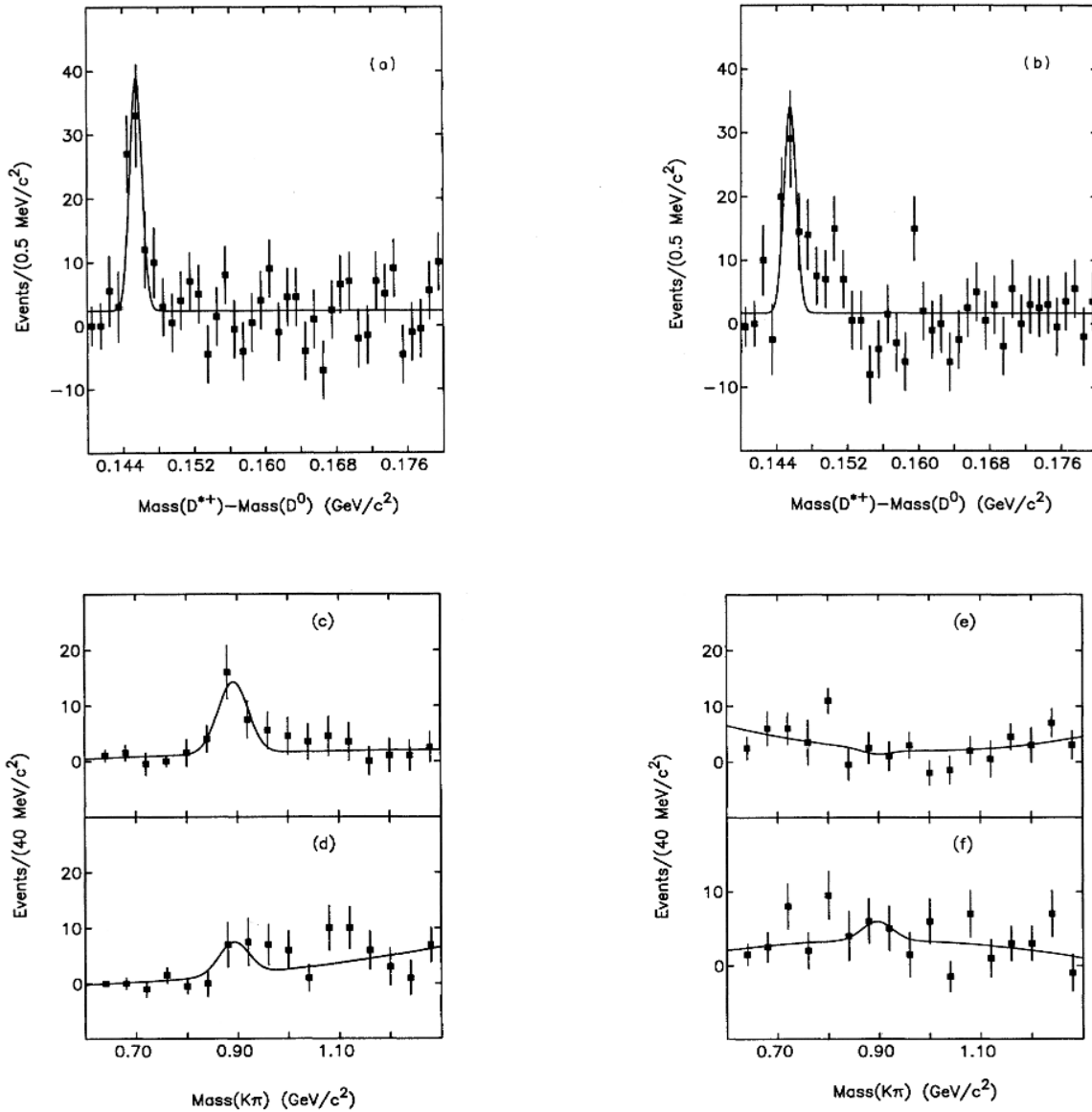


FIG. 6. (a) Background-subtracted  $D^{*+} - D^0$  mass-difference plot, where  $D^0 \rightarrow K^0 K^- \pi^+$ . (b) Background-subtracted  $D^{*+} - D^0$  mass-difference plot, where  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$ . (c) and (d): Background-subtracted  $K\pi$  mass plots for events in the  $D^{*+}$  peak of (a) and (b). (e) shows  $K^0 \pi^+$  and (f) shows  $\bar{K}^0 \pi^-$ . (e) and (f): Background-subtracted  $K\pi$  mass plots for events in the  $D^{*+}$  peak of (a) and (b). (e) shows  $K^- \pi^+$  and (f) shows  $K^+ \pi^-$ .

6(f) shows the  $K^- \pi^+$  invariant mass in  $K_S^0 K^- \pi^+$  events and it does not show a significant peak at the  $\bar{K}^{*0}$ . The results are tabulated in Table I and are in agreement with those of the E-691 Collaboration [6].

#### THE DECAY $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^- \pi^+ \pi^-$

This is a Cabibbo-allowed decay mode, but has a smaller available phase space than lower-multiplicity decay modes such as  $\bar{K}^0 \pi^+ \pi^-$ . The analysis proceeds in a similar method as above, using an  $x_p$  cut of 0.5. This sideband-subtracted mass-difference plot is shown in Fig. 7. It shows a clear signal of  $56 \pm 11$  events. This decay mode is compared to  $\bar{K}^0 \pi^+ \pi^-$  obtained with similar cuts, to obtain a branching ratio  $B(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^- \pi^+ \pi^-) / B(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) = 0.149 \pm 0.026$  (see Table I). This is the most statistically significant observation of this mode. One previous, low-statistics result has been published [5].

#### THE DECAY $D^0 \rightarrow K_S^0 K_S^0 K_S^0$

This decay, first observed by the ARGUS Collaboration [12], is Cabibbo allowed but involves the popping of an  $s\bar{s}$  pair. Like the  $\bar{K}^0 \phi$  mode it cannot be formed from a simple spectator process, and so its existence indicates either  $W$  exchange [Fig. 1(f)] or final-state interactions are taking place. This particular mode is very free of background, but as three neutral  $K$ 's have to be observed, the efficiency is rather low. The  $K_S^0$  cuts used for this decay are loosened compared with the others used in this paper; the flight path of the  $K_S^0$  is required to be greater than 2 mm and the calculated invariant mass to be within 20 MeV/ $c^2$  of the known  $K_S^0$  mass. Furthermore, we do not look for the  $D^*$  decay, but merely plot the observed  $K_S^0 K_S^0 K_S^0$  invariant mass for  $x_p > 0.5$  (Fig. 8). It shows a clear  $D^0$  peak. This mode is compared with  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  analyzed using the same  $x_p$  cut and with

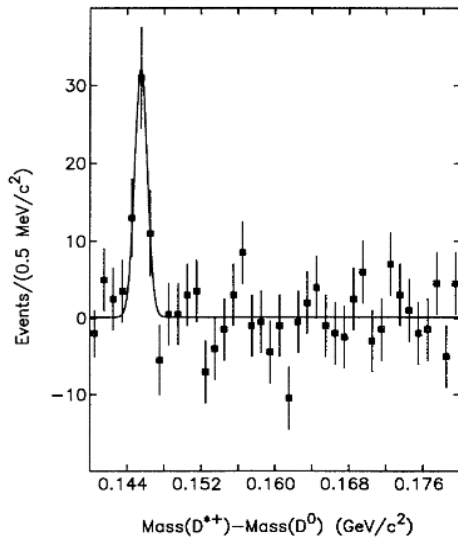


FIG. 7. Background-subtracted  $D^{*+}-D^0$  mass-difference plot, where  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^- \pi^+ \pi^-$ .

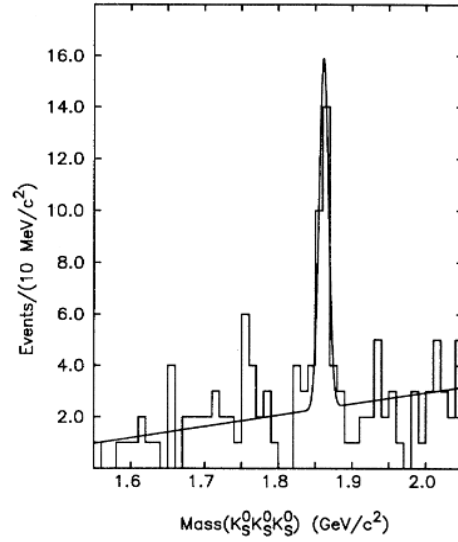


FIG. 8. The  $K_S^0 K_S^0 K_S^0$  invariant-mass plot, with a requirement of  $x_p > 0.5$ .

no  $D^*$  requirement. The result is consistent with the previous observation of this mode [12]. We do not convert this measurement into one for  $D^0 \rightarrow \bar{K}^0 K^0 \bar{K}^0$  as the conversion factor will depend on the mechanism through which the decay proceeds.

#### THE DECAY $D^+ \rightarrow \bar{K}^0 \bar{K}^0 K^+$

This decay mode of the  $D^+$ , which involves the popping of an  $s\bar{s}$  pair, has not been previously observed. The analysis uses the tight  $K_S^0$  definitions as used in the  $\bar{K}^0 K^+ \pi^-$  analysis above, and a requirement of  $x_p > 0.5$ . The  $K_S^0 K_S^0 K^+$  invariant-mass spectrum is shown in Fig. 9, and shows a clear signal in the  $D^+$  region. The yield from this fit is then compared with the well-known decay mode of the  $D^+$  into  $K^- \pi^+ \pi^+$ .

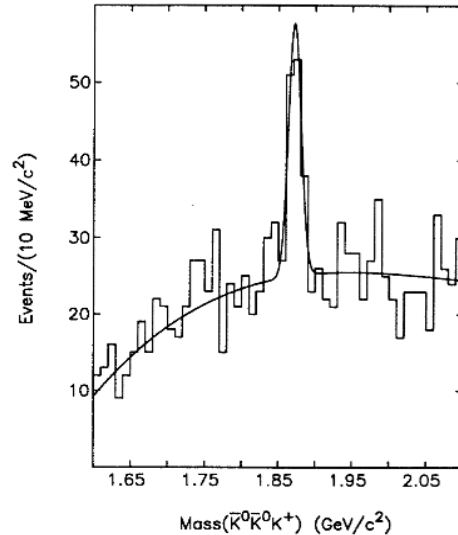


FIG. 9.  $\bar{K}^0 \bar{K}^0 K^+$  invariant-mass plot for combinations with  $x_p > 0.5$ .

### INTERPRETATION OF RESULTS

The results are tabulated in Table I. The absolute branching fractions are obtained by multiplying the branching ratios obtained in this experiment by the relevant world-average absolute branching fractions as calculated by the Particle Data Group [4,13]. The resulting branching fractions are listed in Table II. Systematic uncertainties that have been considered include uncertainties due to the fitting techniques used and uncertainty as to the exact resonant substructure involved leading to an uncertainty in the efficiency calculations. The estimated systematic uncertainty is around 5% for all modes. As statistical uncertainties of the measurements are 13% or greater, the systematic uncertainties have not been included unless specifically mentioned in the text.

### COMPARISON WITH THEORY

Unfortunately, it has proved very difficult to produce reliable theoretical models of charmed-meson decays. Several authors [14–16] have produced lists of expected widths for two-body decays. Bauer *et al.*, do this for a spectator model, which therefore does not predict any decays into  $\phi\bar{K}^0$ . In other modes their model is quite successful, although their value for  $B(D^0 \rightarrow \phi\rho^0)$  is somewhat higher than our measurement. Yu *et al.* parametrize annihilation effects together with the nonfactorizable part of the spectator diagrams and manage reasonable agreement with the two-body decays present-

ed here, including  $\phi\bar{K}^0$ . Both models predict that  $K^{*+}K^-$  should be larger than  $K^{*-}K^+$ , as is found in the data, though statistical uncertainties are still very large. This would indicate that the quarks arising from the virtual  $W^+$  tend to materialize more frequently as a vector particle than do the spectator quarks; the same trend is true for Cabibbo-allowed decays, where  $B(D^0 \rightarrow \bar{K}^-\rho^+)$  is found to be greater than  $B(D^0 \rightarrow K^{*-}\pi^+)$  [4].

The limits on  $K^{*0}\bar{K}^0$  and  $\bar{K}^{*0}K^0$  are lower than the measurement [8], made using the same data, of  $B(D^0 \rightarrow \bar{K}^0 K^0)$  of  $0.13^{+0.07+0.02}_{-0.05-0.02}\%$ . This fact may shed some light as to the mechanism of these decays.

### CONCLUSIONS

We have presented new measurements of the branching fractions of  $D^0$  mesons into  $\pi^+\pi^-\pi^+\pi^-$ ,  $K^+K^-\pi^+\pi^-$ ,  $\bar{K}^0K^-\pi^+$ ,  $K^0K^+\pi^-$ , and  $K_S^0K_S^0K_S^0$  together with some of their resonant substructure. The comparatively large branching fractions into  $K_S^0K_S^0K_S^0$  and  $\bar{K}^0\phi$ , which cannot be explained by a simple spectator model of decay, are a challenge for theoretical modeling including  $W$ -exchange and final-state interactions. We have also observed and measured the previously unseen decay of the  $D^+ \rightarrow \bar{K}^0\bar{K}^0K^+$ . We do not observe the doubly-Cabibbo-suppressed decays  $K^+\pi^-$  and  $K^+\pi^-\pi^+\pi^-$ , but obtain stringent limits on these processes.

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