9-2012

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September 16, 2012

Abstract

We compare the welfare of different combinations of monetary and currency policies in an open-economy macroeconomic model that incorporates two important features of many small economies: a high level of vertical international trade and a prevalent use of a large trade partner's currency as the invoicing currency for both imports and exports. In this environment, a small economy prefers a fixed exchange rate regime over a flexible regime, while the larger economy prefers a flexible exchange rate regime. There are two main causes underlying our results. First, in the presence of sticky prices, relative prices adjust through changes in the exchange rate. Multiple stages of production and trade make it more difficult for one exchange rate to balance the whole economy by adjusting several relative prices throughout the vertical chain of production and trade. Namely, there is a trade-off between delivering an efficient relative price between home and foreign final goods and delivering an efficient relative price between home and foreign intermediate goods. Second, because the small economy uses the larger economy's currency in trade, it faces a high degree of exchange rate pass-through under a flexible regime and hence suffers from the lack of efficient relative prices in vertical trade. The larger economy, however, does not face this problem because its level of exchange rate pass-through is low.

JEL classification: F3, F4

Keywords: vertical production and trade, exchange rate pass-through, monetary policy

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1 Introduction

Many small economies have two prominent features. First, they engage in extensive vertical trade with one or more large economies; that is, these countries trade not only final goods but also a large quantity of intermediate goods. Second, they primarily use the currencies of their large trade partners to trade, such as the US dollar and the Euro. Two important examples are Canada and Mexico, the two trade partners of the US in the North American Free Trade Agreement (NAFTA).

There is a substantial amount of vertical trade between Canada and the US, especially in machinery and equipment, industrial goods and materials, and automotive products. Vertical trade accounted for approximately 30% to 35% of total exports for Canada in 1990, and more importantly, approximately 51% of Canada's export growth between 1970 and 1990 (Hummels et al., 2001). As for Mexico, the vertical specialization share of total exports rose steadily between 1984 and 1997 and reached 32% in 1997, based on data from Mexican maquiladoras (Hummels et al., 2001). From the perspective of the US, vertical trade is also a prominent phenomenon. Multinationals account for over half of US total exports in 1999, and 93% of exports by US parent firms to their foreign manufacturing affiliates are inputs for further processing (Hanson et al., 2005). In addition, the increase in intra-industry trade between the US and NAFTA from 1990 to 2007 was almost entirely due to two-way trade in vertical differentiation (Ekanayake et al., 2009).[1]

The data on trade between the Central and Eastern European Countries (CEEC) versus the European Union (EU) also indicate the importance of vertical trade. The EU is the largest trade partner of the CEECs. In 1998, trade with the EU accounts for 81.8% of CEEC total imports of parts and 79.4% of their total exports of parts.[2] Trade in parts has been the most rapidly growing component of international trade for the CEECs (Kaminski and Ng, 2001).[3] The value of CEEC total trade turnover in parts grew almost threefold from 1993 to 1997, with both exports and imports increasing at a similar pace, and concentrating in three sectors: motor vehicle parts, office machinery,

[1]An alternative measurement of vertical trade is the production-sharing intensity of trade, defined as the ratio of affiliate sales of manufactured goods to the US parent as a share of total manufacturing exports to the US in a country (or region). Based on trade flows between US multinationals and their affiliates and between the US and Mexican maquiladoras, the production-sharing intensity of trade is approximately 50% for Canada and 25% for Mexico in 2003 citepBursteinKurzTesar:2008. When maquiladoras are included, the production-sharing intensity of trade increases dramatically from 25% to 55% for Mexico. As the authors explain in their paper, these data likely understate the degree of production sharing because they capture only intra-firm trade and omit arm's-length production sharing.

[2]In particular, the most important trading partner with the CEECs is Germany, which accounts for 39.4% of CEEC total parts imports and 49.7% of their total parts exports.

[3]Kaminski and Ng (2001) focus on ten CEECs: Czech Republic, Estonia, Hungary, Poland, Slovenia, Bulgaria, Romania, Slovakia, Latvia, and Lithuania.

and telecommunications equipment.

There is plenty of empirical evidence suggesting that small economies use the currencies of their large trade partners for international pricing. This invoicing practice implies that the degree of exchange rate pass-through is higher for these small economies than that of their large trade partners. For instance, the US dollar is the dominant invoicing currency in Canada-US trade. Between 1996 and 1998, 92.8% of Canada's exports to the US are invoiced in US dollars, only 4.8% in Canadian dollars, and 2.4% in other currencies (Donnenfeld and Haug, 2008). Similarly, in the later period of 2002 to 2009, over 95% of Canada's imports from the US are invoiced in US dollars by count and over 80% by value (Goldberg and Tille, 2009). A firm-level survey conducted by the Bank of Canada in 2002 also confirms that the US dollar is dominant in invoicing Canadian exports (Murray et al., 2003).

More generally, Goldberg and Tille (2008), Kamps (2006), and Ligthart and Werner (2010) document that the US dollar is the primary invoice currency choice in flows to and from the US, and there is an increasing role of the euro in the EU and its accession countries. As Bacchetta and van Wincoop (2005) note, when exporters in small open economies face strong international competition in a large foreign market, it is often optimal for these exporters to price in the importer's currency because they have small market shares and/or because the substitutability between products is high. Similarly, Goldberg and Tille (2008) emphasize a "coalescing"effect where exporters try to minimize the movements of their prices relative to their competitors.

Although the literature on international trade and finance have recognized the existence and importance of vertical trade (Feenstra, 1998; Hummels et al., 1998, 2001; Yeats, 2001; Yi, 2003, 2010) and the high level of exchange rate pass-through entailed by the choice of invoicing currency by small open economies (Donnenfeld and Haug, 2008; Goldberg and Tille, 2008, 2009; Murray et al., 2003; Kamps, 2006; Ligthart and Werner, 2010), the implication of these two features on monetary policy and exchange rate regimes has not been formally studied. Intuitively, the economic integration fostered by increased vertical trade makes it more difficult for a flexible exchange rate to balance the whole economy. Moreover, because these small economies use foreign currency, for example the US dollar and the Euro, as the currency of export and import price-setting, they are more exposed to exchange rate fluctuations than countries such as the US. Consequently, they face a higher cost of a flexible exchange rate regime. These factors give rise to the following question: Do the trends of increasing vertical trade and the high level of exchange rate pass-through alter the comparison of economic welfare for these small economies under different monetary systems? In particular, how does a multiple currency system with a flexible exchange rate compare to a currency peg or a common currency system?

To answer these questions, we construct a two-country, open-economy macroeconomic model with vertical trade and asymmetric exchange rate pass-through (i.e., a high level of exchange rate pass-through in the smaller economy and a low level of exchange rate pass-through in the larger economy). There are two stages of production in each country. The final goods sector in each country produces tradable final goods using domestic and imported intermediate goods, while the intermediate goods are produced by labor. The production of each type of good in each country faces a stage-specific and country-specific productivity shock. Because the production of tradable final goods requires both domestic and imported intermediate goods, vertical trade is necessary. We assume that all goods are priced in the currency of the larger economy. To complete the model, we specify the monetary policy as a money supply rule and focus on practical targeting rules. We then base the comparison of several exchange rate regimes, such as unilateral peg, monetary union, and inflation targeting with a floating exchange rate, on the welfare of each country.[4]

Our main result is that when vertical trade and a high level of exchange rate pass-through are present, a fixed exchange rate regime is typically more desirable than a flexible exchange rate regime. This result contrasts sharply against the standard result in the open-economy macroeconomics literature, that open economies should favor a flexible exchange rate regime. There are two key reasons behind our result. First, in the face of vertical trade, there are multiple relative prices that need to be adjusted in response to underlying shocks in different stages of production. When prices are sticky, international relative prices adjust through changes in exchange rates. However, there is only one exchange rate, which makes it difficult to balance the entire economy. Indeed, the economy may face costly spillovers of shocks from one stage of production to another via the vertical chain of production and trade if the exchange rate is flexible. Second, the degree of the exchange rate pass-through plays a crucial role in our model. If there is a high degree of exchange rate pass-through into the economy, the effects of the negative spillovers will be strong. Therefore, the exchange rate risk under a flexible

[4]To facilitate the comparison of our results with those of the standard new open-economy macroeconomics (NOEM) models, we assume that the financial market is complete and that a production subsidy exists in each stage of production. Consequently, we abstract away from distortions caused by incomplete international risk-sharing and monopolistic competition and focus on a single friction: nominal price rigidity.

exchange rate regime will be costly for the smaller country because its exports and imports are both priced in the currency of the larger country. In comparison, the larger country's trade is priced in its own currency; hence the exchange rate does not affect its relative prices and the negative spillovers will not occur. The larger country would then favor a flexible exchange rate regime even when there is vertical trade.

Our paper builds on the new open-economy macroeconomics (NOEM) literature on evaluating monetary policies.[5] A long-standing idea in international economics is that a flexible exchange rate regime is desirable because when prices are sticky, nominal exchange rate movements can adjust the relative prices of home and foreign goods (Friedman, 1953). Obstfeld and Rogoff (1995, 1998, 2000a, 2002) make a similar argument in a dynamic stochastic general equilibrium framework featuring producer currency pricing (PCP). They show that when the monetary authorities respond only to domestic shocks, the flexible exchange rate regime can replicate the flexible price equilibrium, i.e., the optimal outcome. However, later papers show that this result depends on the assumption of the pricing currency for international trade. In particular, Devereux and Engel (2003) show that under local currency pricing (LCP), there is no exchange rate pass-through, and hence little value in exchange rate flexibility. In their model, a fixed exchange rate regime is optimal. However, depending on the model setup and types of shocks, LCP does not always imply that a fixed exchange rate regime is strictly preferred, e.g., Devereux et al. (2006) and Tille (2002). Compared to these papers, we show that when vertical trade is introduced, even a country subject to a high level of exchange rate pass-through may prefer a fixed exchange rate regime.

This paper is also closely related to the branch of international macroeconomics literature that emphasizes the vertical integration of production and trade. Huang and Liu (2001) analyze a closed-economy dynamic stochastic general equilibrium (DSGE) model with sticky prices and multiple stages of production. They find that monetary shocks can generate persistent fluctuations in aggregate output. Their model is extended to study the welfare effect of inflation on the price of intermediate goods and the cross-country correlation on consumption and on output in a two-country model, as well as the welfare impact of a unilateral monetary expansion in an open-economy setup (Huang and Liu, 2005, 2004, 2006). Another article that features vertical production and trade is Shi and Xu (2007), which examines an optimal linear monetary policy rule in a model with vertical trade where

[5]Corsetti and Pesenti (2007) provide a graphical representation of optimal monetary policy in open economy models.

both counties practice PCP.

Although our paper is closely related to Shi and Xu (2007), it differs in important ways. In terms of model setup, ours features asymmetric exchange rate pass-through and focuses on realistic simple targeting rules that do not require the observation of fundamental shocks. Regarding results, although they show that the optimal monetary policy implies lower exchange rate volatility than that of an economy without vertical trade, their results are still consistent with the standard NOEM literature in that a flexible exchange rate regime is optimal. Instead, our paper finds that the fixed exchange rate regime is preferred by the country that faces a high level of exchange rate pass-through. Moreover, if a country does not bear much of the exchange rate risk (i.e., exchange rate changes have a limited effect on its relative prices), then even if vertical trade is present, there is little room for exchange rate policy.

We contribute to the literature by showing that the interaction between vertical trade and the degree of exchange rate pass-through can alter the desirability of a flexible exchange rate regime vs. a fixed exchange rate regime. When there is vertical trade, a small economy generally prefers a fixed exchange rate regime because it uses foreign currency in international pricing, and hence is subject to a high degree of exchange rate pass-through. Meanwhile, a large economy uses its own currency to price exports and imports and is thus subject to a low level of exchange rate pass-through and in general favors a flexible exchange rate regime. Notice that our model is highly stylized to highlight the role played by vertical trade and the degree of exchange rate pass-through. Therefore, our results should not be seen as an outright endorsement for or a prescription of a fixed exchange rate regime in countries that exhibit these characteristics. To evaluate the optimality of an exchange rate regime for a particular country, more factors should be included in the analysis.

The remainder of the paper proceeds as follows. Section 2 describes the structure of the model. Section 3 presents the method used to compute welfare. Section 4 compares alternative monetary policies for the home and foreign countries and highlights the key driving forces of the results. Conclusions follow.

2 Model

In our model, the world has two countries, home and foreign. Each country is populated by households that are immobile across borders. We normalize the world population to a measure of one. Home households reside on the interval $[0, n]$, and foreign households reside on the interval $(n, 1]$. Hence, the parameter n is the size of the home country. Households derive utility from aggregate consumption, real money balance, and leisure. In each country, there are two production stages, one for intermediate and one for final goods. As illustrated in Figure 1, the two countries trade intermediate goods, final goods, and assets, but not labor. For simplicity, we abstract away from any dynamics by considering a single period model with uncertainty and with two sub-periods, one before and the other after the realization of productivity shocks.[6] Figure 2 explains the sequence of events within the period. First, households enter the period and trade in a complete set of state-contingent nominal bonds that specify payoffs in all possible future states. Next, monetary authorities announce monetary policy rules. Firms then set prices in advance, taking into account their expected demands, marginal costs, and discount factors. After shocks are realized, production and consumption take place, and the exchange rate is determined.

Below, we describe the detailed structure of the home economy. Unless indicated otherwise, the foreign economy has an identical structure. Where appropriate, variables of the foreign economy are denoted with an asterisk.

2.1 Household

The representative household in the home country, taking prices and wages as given, maximizes the following expected utility:

$$U = \mathbb{E}\left[ln(C) + \chi ln(\frac{M}{P}) - \eta L\right] \tag{1}$$

where C is the aggregate consumption comprised of all home- and foreign-produced final goods

$$C = \left[n^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

The parameter θ ($\theta > 0$) is the elasticity of substitution between home and foreign final goods. The fraction $\frac{M}{P}$ denotes real money balances and L represents labor supply. The parameters χ and η are positive. There is a continuum of home goods and foreign goods of measure n and $1-n$, respectively.

[6]Our results will hold in an infinite horizon model because of full risk-sharing and complete price stickiness.

Indices of home and foreign produced goods are defined as

$$C_H = \left[(\frac{1}{n})^{\frac{1}{\phi}} \int_0^n C_H(i)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} \tag{3}$$

$$C_F = \left[(\frac{1}{1-n})^{\frac{1}{\phi}} \int_n^1 C_F(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}} \tag{4}$$

where $C_H(i)$ denotes the home final goods of variety i, and $C_F(j)$ is the foreign final goods of variety j. The parameter ϕ ($\phi > 1$) is the elasticity of substitution between varieties of goods within each country. The implied aggregate consumer price index is then

$$P = \left[nP_H^{1-\theta} + (1-n)P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{5}$$

where P_H (P_F) represents the price index for home (foreign) final goods sold in the home country,

$$P_H = \left[\frac{1}{n} \int_0^n P_H(i)^{1-\phi} di \right]^{\frac{1}{1-\phi}} \tag{6}$$

$$P_F = \left[\frac{1}{1-n} \int_n^1 P_F(j)^{1-\phi} dj \right]^{\frac{1}{1-\phi}} \tag{7}$$

Home individual demand for home goods i (foreign goods j) is then given by

$$C_H(i) = \frac{1}{n} [\frac{P_H(i)}{P_H}]^{-\phi} C_H \tag{8}$$

$$C_F(j) = \frac{1}{1-n} [\frac{P_F(j)}{P_F}]^{-\phi} C_F \tag{9}$$

where C_H (C_F) refers to home individual demand for all home (foreign) goods,

$$C_H = n(\frac{P_H}{P})^{-\theta} C \tag{10}$$

$$C_F = (1-n)(\frac{P_F}{P})^{-\theta} C \tag{11}$$

Note that the total demand for home goods is $Y^d = nC_H + (1-n)C_H^*$ and the total demand for foreign goods is $Y^{d*} = nC_F + (1-n)C_F^*$. Moreover, the following identities always hold: $\int_0^n P_H(i)C_H(i)di = P_H C_H$, $\int_n^1 P_F(j)C_F(j)dj = P_F C_F$, and $P_H C_H + P_F C_F = PC$.

Residents of each country can purchase a full set of state-contingent nominal bonds; thus, the household budget constraint is

$$PC + M + \sum_{\xi} q(\xi)B(\xi) = WL + \Pi + B + M_0 + T \tag{12}$$

where $q(\xi)$ and $B(\xi)$ represent the price and the amount of a specific bond that pays one unit of home currency in state ξ, and zero in other states. After shocks are realized, households choose the optimal level of consumption C, money holding M, and labor supply L, while households are financed by labor income WL, payoff from the state-contingent bonds B, profit from all domestic firms Π, the initial money balance M_0, and the government transfer T. We assume that the government repays any seigniorage income through the lump-sum transfer. That is, the government budget constraint is $M_0 = M + T$.

The trade in state-contingent nominal bonds will lead to the following risk-sharing condition

$$\frac{1}{PC} = \Gamma_0 \frac{1}{SP^*C^*} \tag{13}$$

where S is the nominal exchange rate defined as the price of foreign currency in terms of home currency. Γ_0 is state invariant and determined in the initial market for assets.[7] This paper focuses on the effect of vertical trade and asymmetric exchange rate pass-through on the choice of exchange rate regimes. Hence, we simplify the role of financial markets by assuming that there is complete international consumption risk-sharing. This assumption is typical in much of the extant NOEM literature. Furthermore, we assume that the monetary rules are announced after the state-contingent bond markets have closed. We would obtain the same results if we made the reverse assumption because our model is static. In a dynamic model, Γ_0 will become endogenous and depend on policy choices, as shown in Senay and Sutherland (2011). In that case, the timing of asset trade may be crucial for policy analysis, but it is beyond the scope of this paper.

Households' other optimization conditions, namely, the money demand function and the implicit labor supply schedule, are

$$M = \chi PC \tag{14}$$

[7] For detailed derivation of the risk-sharing condition, please see Devereux and Engel (2003) and Chari et al. (2002).

$$W = \eta PC \tag{15}$$

As a result, the nominal wage is proportional to money in circulation. In addition, the optimization conditions imply that the exchange rate is determined by the relative money supply between the home and foreign countries, $S = \Gamma_0 \frac{M}{M^*}$.

2.2 Final Goods Stage

Home final goods producer i has access to the following technology

$$Y(i) = A\left[n^{\frac{1}{\epsilon}} X_H(i)^{\frac{\epsilon-1}{\epsilon}} + (1-n)^{\frac{1}{\epsilon}} X_F(i)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

where A is the stochastic productivity shock at the final goods stage in the home country with $E(logA) = 0$ and $Var(logA) = \sigma_A^2$. The parameter ϵ ($\epsilon > 0$) is the elasticity of substitution between home and foreign intermediate goods. $X_H(i)$ is a basket of home intermediate goods used in the production of home final goods i. $X_F(i)$ is a basket of foreign intermediate goods used in the production of home final goods i. They are defined as

$$X_H(i) = \left[(\frac{1}{n})^{\frac{1}{\phi}} \int_0^n X_H(i,h)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}} \tag{17}$$

$$X_F(i) = \left[(\frac{1}{1-n})^{\frac{1}{\phi}} \int_n^1 X_F(i,f)^{\frac{\phi-1}{\phi}} df\right]^{\frac{\phi}{\phi-1}} \tag{18}$$

where h and f index the varieties of home and foreign intermediate goods. Solving the cost minimization problem of final goods producer i, we obtain the unit cost of producing final goods i

$$\Lambda = \frac{1}{A}\left[n(\tilde{P}_H)^{1-\epsilon} + (1-n)(\tilde{P}_F)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{19}$$

where $\tilde{P}_H$ is the price index of home intermediate goods denominated in home currency, and $\tilde{P}_F$ is the price index of foreign intermediate goods sold in the home country and denominated in home currency.

To characterize the asymmetric exchange rate pass-through observed in the data, we assume that home final goods producers set prices in home currency in the home market, and set prices in foreign currency in the foreign market. Namely, the home final goods producers practice local currency pricing (LCP). In contrast, foreign final goods producers always set prices in the foreign currency in both home and foreign markets. That is, they practice producer currency pricing (PCP). Under these pricing assumptions, we do not expect the law of one price or purchasing power parity (PPP) to hold in general. To abstract away from distortions associated with monopolistic competition in both final goods and intermediate goods, we further assume that in each stage of production, there exists a production subsidy, $\gamma = \frac{1}{\phi-1}$, such that the price markup in each stage equals zero. After introducing such subsidies, the only type of distortion left in the economy is nominal price rigidity.

Given the demand structure and the unit cost of final goods, the home firm i's profit maximization problem is

$$\max \mathbb{E}\pi(i) = \mathbb{E}D\{[(1+\gamma)P_H(i) - \Lambda]n[\frac{P_H(i)}{P_H}]^{-\phi}(\frac{P_H}{P})^{-\theta}C$$
$$+[S(1+\gamma)P_H^*(i) - \Lambda](1-n)[\frac{P_H^*(i)}{P_H^*}]^{-\phi}(\frac{P_H^*}{P^*})^{-\theta}C^*\}$$

where D is the discount factor, $D = \frac{1}{PC}$. The optimal pricing equation of the home final goods i sold in the home market is

$$P_H(i) = \frac{\phi}{(\phi-1)(1+\gamma)} \frac{\mathbb{E}\left[D\Lambda P_H^{\phi}(\frac{P_H}{P})^{-\theta}C\right]}{\mathbb{E}\left[DP_H^{\phi}(\frac{P_H}{P})^{-\theta}C\right]} \tag{20}$$

As all prices are preset, by applying symmetry in varieties of final goods, we obtain the optimal pricing equation for the home final goods sold in the home market

$$P_H = \frac{\phi}{(\phi-1)(1+\gamma)} \frac{\mathbb{E}(D\Lambda P^{\theta}C)}{\mathbb{E}(DP^{\theta}C)} \tag{21}$$

Similarly, we can derive the optimal pricing equation for the home final goods sold in the foreign market

$$P_H^* = \frac{\phi}{(\phi-1)(1+\gamma)} \frac{\mathbb{E}[D\Lambda(P^*)^{\theta}C^*]}{\mathbb{E}[DS(P^*)^{\theta}C^*]} \tag{22}$$

Regarding the foreign final goods producers, in addition to assuming they always set prices in their own currency, we assume that they do not price to market, that is, $P_F(j) = SP_F^*(j)$. Given that the foreign country is much larger than the home country, it is reasonable to assume that foreign goods producers may find it too costly (e.g., extra information, menu and operation costs) to set a separate price for the home market. Therefore, we can write the foreign firm j's profit maximization problem as

$$\max \mathbb{E}\pi^*(j) = \mathbb{E}D\left\{[(1+\gamma)P_F^*(j) - \Lambda^*]\left[n(\frac{SP_F^*(j)}{P_F})^{-\phi}(\frac{P_F}{P})^{-\theta}C + (1-n)(\frac{P_F^*(j)}{P_F^*})^{-\phi}(\frac{P_F^*}{P^*})^{-\theta}C^*\right]\right\}$$

where $D^* = \frac{1}{P^*C^*}$. In a symmetric equilibrium, we have

$$P_F^* = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D^*\Lambda^*[n(\frac{P}{S})^{\theta}C + (1-n)(P^*)^{\theta}C^*]\right\}}{\mathbb{E}\left\{D^*\left[n(\frac{P}{S})^{\theta}C + (1-n)(P^*)^{\theta}C^*\right]\right\}} \tag{23}$$

2.3 Intermediate Goods Stage

Home intermediate goods producer h has the following technology[8]

$$X_H(h) = \tilde{A}L(h) \tag{24}$$

where $\tilde{A}$ is the productivity shock at the intermediate goods stage in the home country with $E(log\tilde{A}) = 0$ and $Var(log\tilde{A}) = \sigma^2_{\tilde{A}}$. From the cost minimization problems of final goods producers, we can obtain the total demands for intermediate goods h of all home final goods producers and all foreign final goods producers, $X_H(h)$ and $X_H^*(h)$, respectively,

$$X_H(h) = [\frac{\tilde{P}_H(h)}{\tilde{P}_H}]^{-\phi}(\frac{\tilde{P}_H}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di \tag{25}$$

$$X_H^*(h) = [\frac{\tilde{P}_H^*(h)}{\tilde{P}_H^*}]^{-\phi}(\frac{\tilde{P}_H^*}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj \tag{26}$$

where $\tilde{P}_H(h)$ is the home-currency price of intermediate goods h in the home country, $\tilde{P}_H^*(h)$ is the foreign-currency price of the intermediate goods h in the foreign country, and $\int_0^n Y(i)di$ and $\int_n^1 Y^*(j)dj$ represent the total output of home and foreign final goods.

As in the case of home final goods producers, we assume that the home intermediate goods producers price in the currencies of the local markets (i.e., LCP). Note that the unit cost of producing

[8] We have assumed that no capital is required in either stage of production. Because there is no saving-investment decision in a one-period model, capital would be given exogenously in our simple model. Although the initial level of capital stock may have some impact on welfare, this is not the interest of this paper.

intermediate goods is given by $W/\tilde{A}$. The profit maximization problem for home intermediate goods firm h can be written as

$$\max \mathbb{E} D\{[(1+\gamma)\tilde{P}_H(h) - \frac{W}{\tilde{A}}][\frac{\tilde{P}_H(h)}{\tilde{P}_H}]^{-\phi}(\frac{\tilde{P}_H}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di$$
$$+[S(1+\gamma)\tilde{P}^*_H(h) - \frac{W}{\tilde{A}}][\frac{\tilde{P}^*_H(h)}{\tilde{P}^*_H}]^{-\phi}(\frac{\tilde{P}^*_H}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\}$$

Again, by exploiting that all prices are preset and applying symmetry across varieties, we obtain the optimal prices of home intermediates in the home and foreign countries

$$\tilde{P}_H = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D\frac{W}{\tilde{A}}(\Lambda)^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di\right\}}{\mathbb{E}\left\{D(\Lambda)^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di\right\}} \tag{27}$$

$$\tilde{P}^*_H = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D\frac{W}{\tilde{A}}(\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\right\}}{\mathbb{E}\left\{DS(\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\right\}} \tag{28}$$

As in the case of foreign final goods producers, we assume that foreign intermediate goods firms price in the foreign currency for both markets (i.e., PCP) and do not engage in pricing to market. Under these assumptions, we can solve for the price of foreign intermediate goods that applies to both the home and foreign markets

$$\tilde{P}^*_F = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{\mathbb{E}\left\{D^*\frac{W^*}{\tilde{A}^*}\left[(\frac{\Lambda}{S})^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di + (\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj\right]\right\}}{\mathbb{E}\left\{D^*[(\frac{\Lambda}{S})^{\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di + (\Lambda^*)^{\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj]\right\}} \tag{29}$$

2.4 Monetary policy

We model monetary policy as a simple money supply rule and analyze a range of simple targeting rules which are easy to implement in practice. In particular, the money supply is targeted on easily observable variables, such as the nominal exchange rate and/or certain price index. The home monetary rule can be written as

$$M = \bar{M}\left(\frac{S}{\bar{S}}\right)^{-\delta_s}\left(\frac{P^X}{\bar{P}}\right)^{-\delta_p} \tag{30}$$

where $\bar{S}$ is the target level of the exchange rate, and $\bar{P}$ is the target level of the home price. δ_s and δ_p indicate the degree to which the home monetary authority attempts to control variations in the exchange rate and in the price index, respectively. If the money supply rule targets a particular price index, it stabilizes this price index; hence, the rule eliminates the relative price distortion in the presence of nominal price rigidities. Because the exchange rate responds endogenously to home and foreign disturbances in this case, such a rule represents a type of active floating exchange rate regime. More precisely, a producer price index (PPI) targeting rule is defined as a rule that replicates the environment of fully flexible prices. Under a PPI targeting rule, although prices are one hundred percent pre-fixed by producers, they are set at a level that is optimal, even after shocks are realized. That is to say, firms will stick to the prices they have chosen even when they have chances to readjust the prices after the state of the economy is revealed. In our model, a price index targeting rule is equivalent to an inflation targeting rule with a zero inflation rate target. The variable P^X in home monetary policy equation (30) represents the "flexible"home price; that is, the price that would prevail when prices are flexible. For instance, if the home monetary authority targets the PPI of intermediate goods, then $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\frac{W}{A}$. Instead, if the home monetary authority targets the PPI of final goods, then $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda$.

It is well discussed in the NOEM literature that in a typical complete market open economy without vertical trade, it is optimal to stabilize the PPI inflation rate rather than the CPI inflation rate. In this paper, we look at both the final goods and the intermediate goods PPI inflation targeting because in the presence of vertical trade and production, it is not obvious which PPI should be the target. Our main findings hold under both types of price targeting rules. However, from the welfare point of view, targeting the intermediate goods PPI is much more efficient than targeting the final goods PPI. To save space, we focus on the case of targeting the intermediate goods PPI inflation in the main text, and we detail the case of targeting the final goods PPI inflation in the Appendix.

We will analyze three combinations of monetary regimes that we name from the perspective of the home country. These regimes are (1) the "inflation targeting"regime, in which both home and foreign monetary authorities target the PPI of intermediate goods (i.e., $\delta_s = \delta_s^* = 0$ and $\delta_p = \delta_p^* \to \infty$); (2) the "unilateral peg"regime, in which the home country targets the exchange rate (i.e., $\delta_s \to \infty,\ \delta_p = 0$), while the foreign country targets the PPI of intermediate goods ($\delta_s^* = 0,\ \delta_p^* \to \infty$);[9] (3) a coordinated monetary policy regime, the "monetary union", in which the central bank of the

[9]We ignore the reverse of case (2), that is, the foreign country pegs unilaterally to the home currency. This is because the foreign economy is much larger in size, and it would not be optimal for the foreign country to engage in a unilateral peg.

monetary union, acting as a social planner, maximizes world welfare subject to an inflation targeting rule and the constraint that the exchange rate must be kept constant at all times. Specifically, the common central bank maximizes

$$nU + (1-n)U^* \tag{31}$$

subject to

$$\begin{aligned} & n[PC + M + \sum_{\xi} q(\xi)B(\xi)] + (1-n)[SP^*C^* + SM^* + \sum_{\xi} q(\xi)B^*(\xi)] \\ & = n[WL + \Pi + B + M_0 + T] + (1-n)[SW^*L^* + S\Pi^* + B^* + SM_0^* + ST^*] \end{aligned} \tag{32}$$

$$S \equiv 1 \tag{33}$$

$$M \equiv nM + (1-n)M^* \;=\; \bar{M}\left(\frac{P^X}{\bar{P}}\right)^{-\delta_p} \tag{34}$$

where the common central bank targets a weighted average of all the producer prices of intermediate goods; that is, the price target is $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\left(\frac{W}{\tilde{A}}\right)^n\left(\frac{W^*}{\tilde{A}^*}\right)^{1-n}$ and the policy parameter on the price target δ_p approaches positive infinity. [10]

2.5 Market Clearing

The final goods market clearing conditions are

$$\int_0^n Y(i)di = \left[n^2\left(\frac{P_H}{P}\right)^{-\theta} C + n(1-n)\left(\frac{P_H^*}{P^*}\right)^{-\theta} C^*\right] \tag{35}$$

$$\int_n^1 Y^*(j)dj = \left[n(1-n)\left(\frac{P_F}{P}\right)^{-\theta} C + (1-n)^2\left(\frac{P_F^*}{P^*}\right)^{-\theta} C^*\right] \tag{36}$$

The intermediate goods market clearing conditions are

$$\tilde{A}L = (\frac{\tilde{P}_H}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di + (\frac{\tilde{P}_H^*}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj \tag{37}$$

$$\tilde{A}^*L^* = (\frac{\tilde{P}_F}{\Lambda})^{-\epsilon}\frac{1}{A^{1-\epsilon}}\int_0^n Y(i)di + (\frac{\tilde{P}_F^*}{\Lambda^*})^{-\epsilon}\frac{1}{(A^*)^{1-\epsilon}}\int_n^1 Y^*(j)dj \tag{38}$$

[10] Benigno (2004) shows that if two regions in a currency union have the same degree of nominal rigidity, it is optimal for the central bank to target the average inflation rate weighted by the economic sizes of the two regions. In practice, the European Monetary Union targets the average CPI of member countries, weighted by their consumption shares.

2.6 Equilibrium

The *equilibrium* comprises a set of prices $\{P,\, P^*,\, P_H,\, P_H^*,\, P_F^*,\, \tilde{P}_H,\, \tilde{P}_H^*,\, \tilde{P}_F^*,\, \Lambda,\, \Lambda^*,\, W,\, W^*,\, S\}$ and a set of quantities $\{C,\, C^*,\, L,\, L^*,\, M,\, M^*\}$, which solve a system of equations (5), (13)-(15), (19), (30), and their foreign counterparts as well as (21)-(23), (27)-(29) and (37)-(38), given the productivity shocks at each stage of production, A, A^*, $\tilde{A}$, and $\tilde{A}^*$.

3 Solving the Model

Because it is not possible to derive an exact analytical expression for welfare in this model, and a second-order accurate solution is required for welfare comparison,[11] we solve the model by computing the second-order approximation around the non-stochastic steady state.

The second-order approximation of the welfare measure is given by

$$\tilde{U} \;=\; E\left[\hat{C} - \eta\bar{L}\left(\hat{L} + \frac{1}{2}\hat{L}^2\right)\right] + O(\epsilon^3) \tag{39}$$

where $\tilde{U}$ is the deviation of welfare from the non-stochastic equilibrium. $\bar{L}$ is the steady-state values of labor supply, and $\eta\bar{L} = 1$. Hereafter, we use the Jonesian hats to denote the log-deviation of a variable x from its steady-state value $\bar{x}$, that is, $\hat{x} = log(x) - log(\bar{x})$. The term $O(\epsilon^n)$ represents residuals of an equation approximated to an order of $n-1$. Welfare is increasing in the expected level of consumption but is decreasing in the expected level of labor and in the variance of labor.

The second-order approximations of the consumption price index in each country are given by

$$\hat{P} \;=\; n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + \lambda_P + O(\epsilon^3) \tag{40}$$

$$\hat{P}^* \;=\; n\hat{P}_H^* + (1-n)\hat{P}_F^* + \lambda_{P^*} + O(\epsilon^3) \tag{41}$$

where λ_P and λ_{P^*} collect the second-order terms

$$\lambda_P \;=\; \frac{n(1-n)(1-\theta)}{2}(\hat{P}_H - \hat{P}_F^* - \hat{S})^2 \tag{42}$$

$$\lambda_{P^*} \;=\; \frac{n(1-n)(1-\theta)}{2}(\hat{P}_H^* - \hat{P}_F^*)^2 \tag{43}$$

[11] The need for a second-order accurate solution in welfare analysis has been well addressed in the literature. For example, see Collard and Juillard (2001), Kim and Kim (2003), and Schmitt-Grohe and Uribe (2004).

We define terms of trade as the price of exports in terms of imports. Therefore, $E(\lambda_P)$ ($E(\lambda_{P^*})$) represents the variance of the final goods terms of trade for the home (foreign) country.

Similarly, the second-order approximations of the unit cost of production of final goods in each country are

$$\hat{\Lambda} = -\hat{A} + n\hat{\tilde{P}}_H + (1-n)(\hat{\tilde{P}}_F^* + \hat{S}) + \lambda_\Lambda + O(\epsilon^3) \tag{44}$$

$$\hat{\Lambda}^* = -\hat{A}^* + n\hat{\tilde{P}}_H^* + (1-n)\hat{\tilde{P}}_F^* + \lambda_{\Lambda^*} + O(\epsilon^3) \tag{45}$$

where λ_Λ and λ_{Λ^*} summarize the second-order terms

$$\lambda_\Lambda = \frac{n(1-n)(1-\epsilon)}{2}(\hat{\tilde{P}}_H - \hat{\tilde{P}}_F^* - \hat{S})^2 \tag{46}$$

$$\lambda_{\Lambda^*} = \frac{n(1-n)(1-\epsilon)}{2}(\hat{\tilde{P}}_H^* - \hat{\tilde{P}}_F^*)^2 \tag{47}$$

and $E(\lambda_\Lambda)$ ($E(\lambda_{\Lambda^*})$) represents the variance of the intermediate goods terms of trade for the home (foreign) country.

The second-order approximations of the optimal pricing equations are given by

$$\hat{P}_H = E(\hat{\Lambda}) + \lambda_{P_H} + O(\epsilon^3) \tag{48}$$

$$\hat{P}_H^* = E(\hat{\Lambda} - \hat{S}) + \lambda_{P_H^*} + O(\epsilon^3) \tag{49}$$

$$\hat{P}_F^* = E(\hat{\Lambda}^*) + \lambda_{P_F^*} + O(\epsilon^3) \tag{50}$$

$$\hat{\tilde{P}}_H = E(\hat{W} - \hat{\tilde{A}}) + \lambda_{\tilde{P}_H} + O(\epsilon^3) \tag{51}$$

$$\hat{\tilde{P}}_H^* = E(\hat{W} - \hat{\tilde{A}} - \hat{S}) + \lambda_{\tilde{P}_H^*} + O(\epsilon^3) \tag{52}$$

$$\hat{\tilde{P}}_F^* = E(\hat{W}^* - \hat{\tilde{A}}^*) + \lambda_{\tilde{P}_F^*} + O(\epsilon^3) \tag{53}$$

where λ_{P_H}, $\lambda_{P_H^*}$, $\lambda_{P_F^*}$, $\lambda_{\tilde{P}_H}$, $\lambda_{\tilde{P}_H^*}$, and $\lambda_{\tilde{P}_F^*}$ collect the second-order terms and represent the risk premium that firms build into the corresponding pricing decisions

$$\lambda_{P_H} = E\{\frac{1}{2}\hat{\Lambda}^2 + \hat{\Lambda}(\theta-1)\hat{P}\} \tag{54}$$

$$\lambda_{P_H^*} = E\{\frac{1}{2}(\hat{\Lambda} - \hat{S})^2 + (\hat{\Lambda} - \hat{S})(\theta-1)\hat{P}^*\} \tag{55}$$

$$\lambda_{P_F^*} = E\{\frac{1}{2}\hat{\Lambda}^{*2} + \hat{\Lambda}^*[n(\theta-1)(\hat{P} - \hat{S}) + (1-n)(\theta-1)\hat{P}^*]\} \tag{56}$$

$$\lambda_{\tilde{P}_H} = E\{\frac{1}{2}(\hat{W}-\hat{\tilde{A}})^2 + (\hat{W}-\hat{\tilde{A}})\{\epsilon\hat{\Lambda} - (1-\epsilon)\hat{A} + n[-\theta\hat{P}_H + (\theta-1)\hat{P}] + (1-n)[-\theta\hat{P}^*_H + (\theta-1)\hat{P}^* - \hat{S}]\}\} \quad (57)$$

$$\lambda_{\tilde{P}^*_H} = E\{\frac{1}{2}(\hat{W}-\hat{\tilde{A}})^2 - \frac{1}{2}\hat{S}^2 + (\hat{W}-\hat{\tilde{A}}-\hat{S})\{\epsilon\hat{\Lambda}^* - (1-\epsilon)\hat{A}^* + n[-\theta\hat{S} + (\theta-1)\hat{P}] + (1-n)[(\theta-1)\hat{P}^* - \hat{S}]\}\} \quad (58)$$

$$\lambda_{\tilde{P}^*_F} = E\{\frac{1}{2}(\hat{W}^*-\hat{\tilde{A}}^*)^2 + (\hat{W}^*-\hat{\tilde{A}}^*)\{n[\epsilon(\hat{\Lambda}-\hat{S}) - (1-\epsilon)\hat{A}] + n^2[-\theta\hat{P}_H + \hat{S} + (\theta-1)\hat{P}] + n(1-n)[-\theta\hat{P}^*_H + (\theta-1)\hat{P}^*] + (1-n)[\epsilon\hat{\Lambda}^* - (1-\epsilon)\hat{A}^*] + n(1-n)[-\theta\hat{P}_F + \hat{S} + (\theta-1)\hat{P}] + (1-n)^2[-\theta\hat{P}^*_F + (\theta-1)\hat{P}^*]\}\} \quad (59)$$

Eventually, we can express the approximated welfare function (39) in terms of the second moments of variables in the model and compute the second-order accurate second moments from the first-order solutions for the realized values of endogenous variables.[12] The Appendix documents the complete log-linearized system.

4 Results

To compute welfare, we set the size of the home country to be one-ninth of the foreign country in terms of size, i.e., $n = 0.1$, to match the sizes of the Canadian economy and the American economy.[13] The standard deviation of all productivity shocks is 1% of their steady-state level, i.e., $\sigma^2_A = \sigma^2_{A^*} = \sigma^2_{\tilde{A}} = \sigma^2_{\tilde{A}^*} = 0.0001$, and all shocks are independent of each other. We rely on graphs to present our results. Because the elasticity of substitution between home and foreign intermediate goods (ϵ) and the elasticity of substitution between home and foreign final goods (θ) are important for the transmission of productivity shocks across the border, we explore a number of parameter ranges and combinations.

As reviewed by Obstfeld and Rogoff (2000b), the literature of international economics offers a wide range of estimations for θ from 1.2 to 21.4. The number often used in macroeconomics

[12] This solution technique has been employed in a series of papers; for instance, see Sutherland (2004) and Senay and Sutherland (2005).

[13] The size of the home country does not affect our main results. The results of the case $n = 0.25,\ 0.5$ are qualitatively the same and available upon request. The key reason that the home country prefers a fixed exchange rate regime while the foreign country prefers a flexible exchange rate regime is that within a vertical trade structure, the degree of exchange rate pass-through is high to the home country but low to the foreign country. We will discuss this in greater detail below. It is important to note that this high level of exchange rate pass-through to the home economy originates from the fact that a small economy tends to use foreign currency in international trade pricing. Therefore, strictly speaking, size matters through an indirect channel. Based on the theory of currency pricing in Bacchetta and van Wincoop (2005) and existing empirical observations, we simply assume the smaller economy uses the currency of the larger economy as the pricing currency.

studies is between 1 and 2, e.g., Backus et al. (1994) and Chari et al. (2002), while the literature on international trade provides much larger estimates of this parameter, often around 10.[14] A value smaller than unity is seldom used. Meanwhile, available empirical evidence Saito (2004) suggests that the international substitutability among intermediate goods tends to be much higher than that among final goods, that is, $\epsilon > \theta > 1$. Hence, we consider the relevant parameter range to be $\epsilon,\ \theta > 1$.

4.1 Home Welfare

Figures 3 to 6 plot the home welfare under three combinations of monetary regimes, i.e., inflation targeting, unilateral peg, and monetary union, against θ in the range of $[0, 25]$, while fixing ϵ at $0.5,\ 1,\ 2,\ and\ 5$, respectively. Similarly, Figures 7 to 10 plot home welfare under three combinations of monetary regimes against ϵ in the range of $[0, 25]$, while fixing θ at $0.5,\ 1,\ 2,\ and\ 5$, respectively.

4.1.1 Unilateral Peg vs. Inflation Targeting

From Figures 3 to 6, we can see that home welfare under the unilateral peg is not affected by the elasticity of substitution between home and foreign final goods (θ) because a fixed exchange rate eliminates all relative price changes when prices are sticky. Meanwhile, under inflation targeting, welfare decreases in θ. This is because targeting the producer price of intermediate goods in both countries allows for efficient relative price adjustments at the intermediate goods stage, but not at the final goods stage. When home and foreign final goods become more substitutable (i.e., θ increases), the cost of lacking an efficient relative price adjustment at the final goods stage rises, making the floating exchange rate regime less attractive.

As indicated in Figure 7 to Figure 10, the home welfare under the unilateral peg increases with the elasticity of substitution between home and foreign intermediate goods (ϵ). This is because the pricing risks are always higher for home intermediate goods producers than for foreign intermediate goods producers, meaning that foreign intermediate goods are constantly cheaper than home intermediate goods in terms of price levels, although they are not always lower in terms of relative prices. In the case of a unilateral peg, while foreign intermediate goods prices are optimal because they replicate

[14] Ruhl (2008) reconciles the difference by suggesting that in the international macroeconomics literature, the smaller values of θ correspond to the responses of quantities to transitory shocks, while the larger estimates in the trade literature often rely on responses of quantities to permanent changes in tariff and trade cost.

the flexible prices, home intermediate goods prices are much higher due to a complete absence of relative price adjustments. When the relative price of intermediate goods is fixed due to a fixed exchange rate and nominal price rigidity, the higher the substitutability between home and foreign intermediate goods (i.e., ϵ increases), the lower the expected demand for domestic intermediate goods, which reduces the expected level of labor. In other words, when home and foreign intermediate goods become more substitutable, final goods firms can produce the same amount of final goods using a more effective combination of home and foreign intermediate goods. On average, home households are expected to work less, which explains why the welfare of the unilateral peg is an increasing function of ϵ. This result can also be observed in Figure 3 to Figure 6. There, as ϵ rises, the welfare of the unilateral peg goes up for any given value of θ.

Under the inflation targeting regime, the home welfare has an interesting inverse U-shaped relationship with ϵ, the elasticity of substitution between home and foreign intermediate goods. This inverse U-shaped relationship occurs mainly because there are two opposite effects at work. The first is the relative price effect. As home and foreign intermediate goods become more substitutable, the benefit of having an effective relative price adjustment at the intermediate goods stage increases, which raises the welfare of the inflation targeting regime. The second is a negative spillover effect associated with the feature of vertical production and trade. In this case, as the monetary authority targets PPI of intermediate goods, the exchange rate responds only to productivity shocks at the intermediate goods stage. Consequently, any movement in the exchange rate can cause an inefficient relative price adjustment for the final goods. For instance, suppose there is a positive productivity shock to the production of home intermediate goods. This positive shock leads to the depreciation of the home currency. Although prices are sticky, the depreciation of home currency helps to improve efficiency by lowering the relative price of home intermediate goods. Nevertheless, this depreciation of home currency also makes home final goods cheaper. The world demand for final goods shifts toward those made in the home country, although there is no real fundamental change in the final goods sector. To meet the additional demand with the same technology, home final goods producers must use more intermediate goods, thus causing an excessive demand for the relatively less expensive home intermediate goods. To produce more home intermediate goods, home households must supply more labor, resulting in a decrease in their welfare. The negative spillover effect tends to dominate the relative price effect when home and foreign intermediate goods become more substitutable. Therefore, as ϵ increases, the welfare of the inflation targeting regime first rises and then drops. Moreover, this inverse U-shaped relationship is robust to the value of θ.

There is a third effect - the pricing risk effect - that generates the inverse U-shaped relationship. Under inflation targeting, there is absolutely no pricing risk when home intermediate goods producers sell in the home market. Nor is there pricing risk when foreign intermediate goods producers sell in both home and foreign markets because they practice PCP. Nevertheless, owing to LCP, there are still some pricing risks for home intermediate goods producers selling in the foreign market. As a result, home intermediate goods are more expensive than foreign intermediate goods in terms of price levels. Similar to the argument outlined in the paragraph above, given the relative price of intermediate goods, the higher the substitutability between home and foreign intermediate goods, the higher is the welfare of home households, as there is less demand for home intermediate goods, that is, less work for home households. Under the inflation targeting regime, however, this pricing risk effect has limited influence on welfare because when the exchange rate is flexible, the relative price of intermediate goods varies with the underlying shocks in the economy.

These results show that, in general, for the home country, the unilateral peg dominates the inflation targeting regime, and this result is robust to the value of θ and ϵ.[15]. We can understand the results by focusing on the role of the nominal exchange rate in adjusting relative prices. In the face of external shocks, efficient production and consumption requires adjustment in multiple relative prices throughout the vertical chain of production and trade. However, with nominal prices of intermediate and final goods pre-fixed, one exchange rate cannot achieve efficient adjustments in multiple relative prices simultaneously. Therefore, when prevalent nominal price rigidities and multiple relative prices exist, a floating exchange rate regime coupled with inflation targeting has only a limited ability to stabilize the entire economy.

4.1.2 Unilateral Peg vs. Monetary Union

There is little difference in home welfare under the monetary union in comparison to that under the unilateral peg, although the unilateral peg often dominates the monetary union. Because the exchange rate is fixed under both regimes, home welfare under the monetary union is also not affected by the elasticity of substitution between home and foreign final goods (θ), as shown in Figure 3 to

[15]As shown in Figure 3, it is possible that the inflation targeting regime dominates the unilateral peg when both θ and ϵ are very small. However, those are not likely to be the empirically relevant cases.

Figure 6. Meanwhile, under the monetary union, the common central bank targets a weighted average of home and foreign intermediate goods. Neither country's intermediate goods prices are optimal. Compared to the unilateral peg, pricing risks for the home (foreign) intermediate goods producers are smaller (larger) in the currency union. However, the reduction (increase) in the pricing risks for home (foreign) intermediate goods producers is very limited because the home country is much smaller than the foreign country. The common central bank gives a much higher weight to the foreign intermediate goods prices. As a result, home intermediate goods still cost more than their foreign counterparts in absolute terms under the monetary union. For the same reason as discussed in Section 4.1.1, home welfare under the monetary union is again positively related to the elasticity of substitution between home and foreign intermediate goods (ϵ), as shown in Figure 7 - Figure 10. Basically, given the relative price of intermediate goods (i.e., due to single currency and nominal price rigidity), the higher the substitutability between home and foreign intermediate goods, the lower the weight for home intermediate goods in the optimal basket of all intermediate goods used to produce final goods. Therefore, home households are expected to work less, which leads to an improvement in their welfare. This result can also be found in Figure 3 to Figure 6. As ϵ rises, the welfare of the monetary union goes up for any given value of θ.

As previously discussed, the only difference between the monetary union and the unilateral peg is the risk premium included in the pricing of intermediate goods. From the unilateral peg to the monetary union, the pricing risks for home (foreign) intermediate goods producers fall (rise). Accordingly, home intermediate goods become relatively cheaper than foreign intermediate goods; that is, the positive gap between home and foreign intermediate goods prices declines. As the world demand shifts from the foreign to the home intermediate goods, two contrasting effects are at work. On the one hand, as home income rises, home households consume more. On the other hand, home households have to work more, and this effect becomes stronger as home and foreign intermediate goods become more substitutable. From the home country's point of view, the unilateral peg tends to dominate the monetary union, especially for values of ϵ that are more empirically relevant, that is, $\epsilon > 1$.

Putting the results in Sections 4.1.1 and 4.1.2 together, we conclude that for the home country, the unilateral peg generally ranks higher than both the inflation targeting regime and the monetary union.

4.2 Foreign Welfare

Figures 11 to 18 compare the foreign welfare under the three combinations of monetary regimes, that is, inflation targeting, unilateral peg, and monetary union, for different values of θ and ϵ. For the same reasons discussed in Section 4.1, foreign welfare under the unilateral peg and the monetary union is not affected by θ, and the welfare under the inflation targeting is negatively related to θ. The relationships between welfare and θ in the foreign country mirror those in the home country. The foreign country differs in the relationship between ϵ and the welfare under various monetary regimes relative to the home country. With regard to the ranking of the three policy combinations, the foreign country's preference is exactly the opposite of that in the home country. Specifically, for the foreign country, inflation targeting ranks the highest and a monetary union ranks second.

We already know that, regardless of the combination of regimes that is in place, foreign intermediate goods are always cheaper than home intermediate goods in terms of price levels. Therefore, given the relative price of intermediate goods, the higher the substitutability between home and foreign intermediate goods, the lower the welfare of foreign households, as they have to work more to meet the shift in demand. This echoes our previous findings with regard to home households. The negative effect of ϵ on foreign welfare (i.e., the price level effect), present in all three regimes, explains why the foreign welfares under the unilateral peg and under the monetary union are decreasing functions of ϵ. It also accounts for the initial drop in foreign welfare under inflation targeting; in other words, the first part of the U-shape between welfare and ϵ. Meanwhile, because the positive effects of the exchange rate on the relative price adjustments between home and foreign intermediate goods (i.e., the relative price effect) increases with ϵ, as discussed in Section 4.1.1, foreign welfare under inflation targeting eventually increases with ϵ, explaining the second part of the U-shaped relationship. Note that there is no negative spillover effect for the foreign country due to LCP.

For the foreign country, the welfare ranking between the unilateral peg and the monetary union depends on the differences in the relative price between home and foreign intermediate goods. The larger the gap between home and foreign intermediate goods prices, the more labor the foreign households must supply. Consequently, the monetary union dominates the unilateral peg in the foreign country as long as ϵ is greater than $1/2$. The elasticity ϵ plays a role because foreign households also expect to consume more through complete risk-sharing as we move from the unilateral peg to the monetary union. However, this effect is very small.

The ranking between the monetary union and inflation targeting for the foreign country depends on both θ and ϵ. Intuitively, if home and foreign final goods are highly substitutable while the substitutability between home and foreign intermediate goods is poor, that is, high θ but low ϵ, the benefit of having efficient relative price adjustments at the intermediate goods stage is limited. In this case, it is possible for the monetary union to outperform the inflation targeting, as illustrated in Figure 11 to Figure 14. As discussed above, the most relevant range of the parameters is $\epsilon > theta > 1$.

Overall, our results suggest that for the foreign country, the inflation targeting regime tends to dominate both the unilateral peg and the monetary union.

4.3 Discussion

Our paper finds that under these conditions, a small economy will generally prefer to peg its currency to its large trade partner when the small economy is subject to vertical trade and a high degree of exchange rate pass-through. This result contrasts sharply with those of the standard models in the NOEM literature, which often favor a flexible exchange rate regime.

To further understand the result, we conduct a simple experiment in which we shut down the productivity shocks to the final goods sector. To be specific, we set $\sigma_A^2 = \sigma_{A^*}^2 = 0$ and $\sigma_{\tilde{A}}^2 = \sigma_{\tilde{A}^*}^2 = 0.0001$. As a result, the only source of distortion left in the economy is the sluggish price adjustment at the intermediate goods stage. In this setup, with no shocks to the final goods production, one may expect the key findings of the standard models in the NOEM literature to hold; that is, both countries would prefer to target the PPI of the intermediate goods. However, even with the shocks to the final goods neutralized, we find no qualitative change to our earlier results. To be precise, the home country still prefers a fixed exchange rate regime, while the foreign country mostly prefers an inflation targeting regime.[16]

In this simple experiment, a combination of vertical trade and high exchange rate pass-through once more generates inefficient spillovers from the intermediate goods sector to the final goods sector, even when intermediate goods prices are optimal and there are no shocks at the final goods stage to be mitigated. Specifically, similar to the discussion in Section 4.1, a flexible exchange rate responds to the intermediate goods sector's productivity shocks, creating changes in the relative price of final

[16]See the Appendix for detailed figures.

goods for the home households that must buy foreign goods priced in terms of the foreign (producer) currency. Such changes in the relative price of final goods are undesirable because there are no shocks to the final goods sector. Meanwhile, the foreign country is immune from these negative spillovers because home goods are priced in the foreign (local) currency in the foreign market.

We should note that both features (i.e., vertical trade and high exchange rate pass-through) are necessary conditions for obtaining our results. To draw an analogy, the vertical trade structure creates the bridge for inefficient spillovers, but it is the vehicle of exchange rate that delivers these negative spillovers. If a country has a limited exchange rate pass-through, then even if the bridge of vertical trade is there, no substantial spillovers will arrive. This is exactly the case for the foreign country. However, for countries that have a significant amount of trade priced in foreign currencies, it is subject to the full force of the negative spillovers which can generate serious economic cost associated with a flexible exchange rate regime. Similarly, the feature of high pass-through (for one country) itself is not enough to make the fixed exchange rate regime preferred to the flexible exchange rate regime. Devereux et al. (2006) show that, in a model with asymmetric exchange rate pass-through but without vertical trade, a fixed exchange rate regime is not strictly preferred. Intuitively, when there is no vertical trade, it is possible for a flexible exchange rate regime to achieve efficient adjustment in international relative prices in a single sector.

The essence of our findings is a real tradeoff between the cost and benefit of a flexible exchange rate regime in an environment of vertical trade and asymmetric exchange rate pass-through. Specifically, the cost arises because shocks spill over across sectors in a system of vertical production and trade, making it difficult for a single flexible exchange rate to balance all international relative prices. We find that for the small economies characterized in our model, this cost outweighs the benefit of having efficient relative price adjustments in the intermediate goods stage. Under these conditions, it is possible to make a case for the fixed exchange rate regime for these small economies.

5 Conclusion

In the literature regarding monetary policy for open economies, a flexible exchange rate regime is often considered optimal because the flexible exchange rate can achieve efficient adjustment of international relative prices when prices are sticky. In this paper, we take into account two important features of many small economies: vertical trade and the use of foreign currencies as pricing currencies. The first feature implies there are multiple relative prices in international trade that are associated with different stages of production. The second feature implies that the level of exchange rate pass-through is high for these small economies but low for their large trade partners. In this environment, it is difficult for a single exchange rate to adjust all relative prices efficiently, and the difference in exchange rate pass-through leads to different exposures to exchange rate risk. We show that the welfare comparison between flexible and fixed exchange rate regimes depends on the interactions between vertical trade and the degree of exchange rate pass-through. In particular, a small economy favors a fixed exchange rate regime over a flexible exchange rate regime, while its large trade partner generally favors a flexible regime.

We should caution that the purpose of the paper is to highlight the important welfare effects of vertical trade and asymmetric exchange rate pass-through under different exchange rate regimes. Our results do not imply that small economies with these features should unquestionably adopt a fixed exchange rate, as the choice of monetary and exchange rate regimes should be based on a thorough consideration of various economic and political factors.

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Figure 1: Structure of the Model

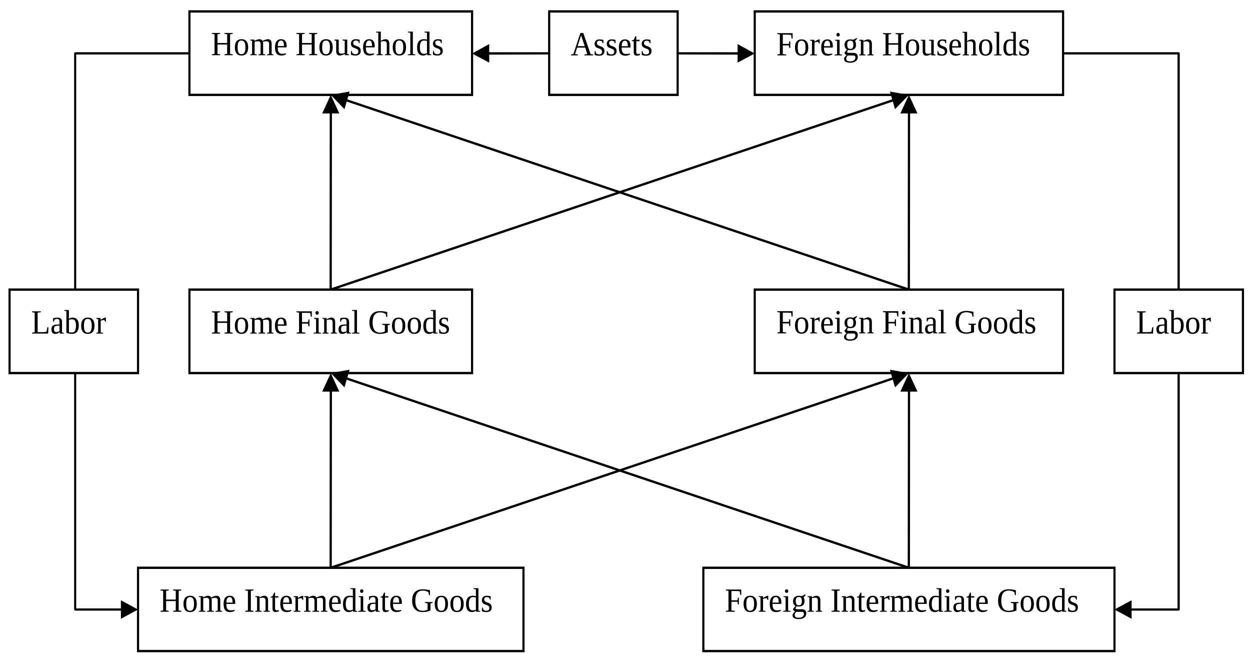


Figure 2: Timing of the Model

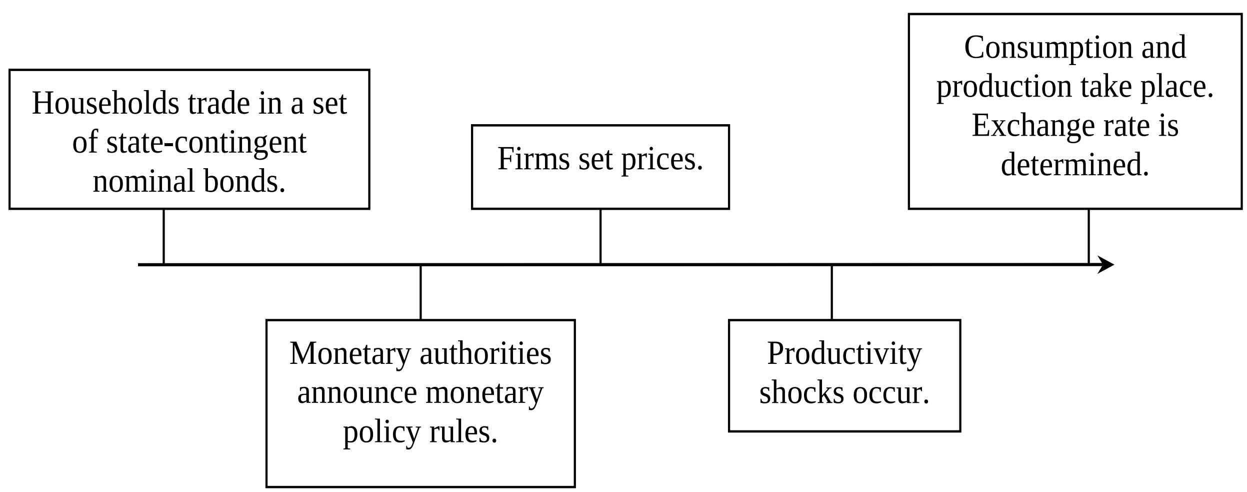


Figure 3: Home Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

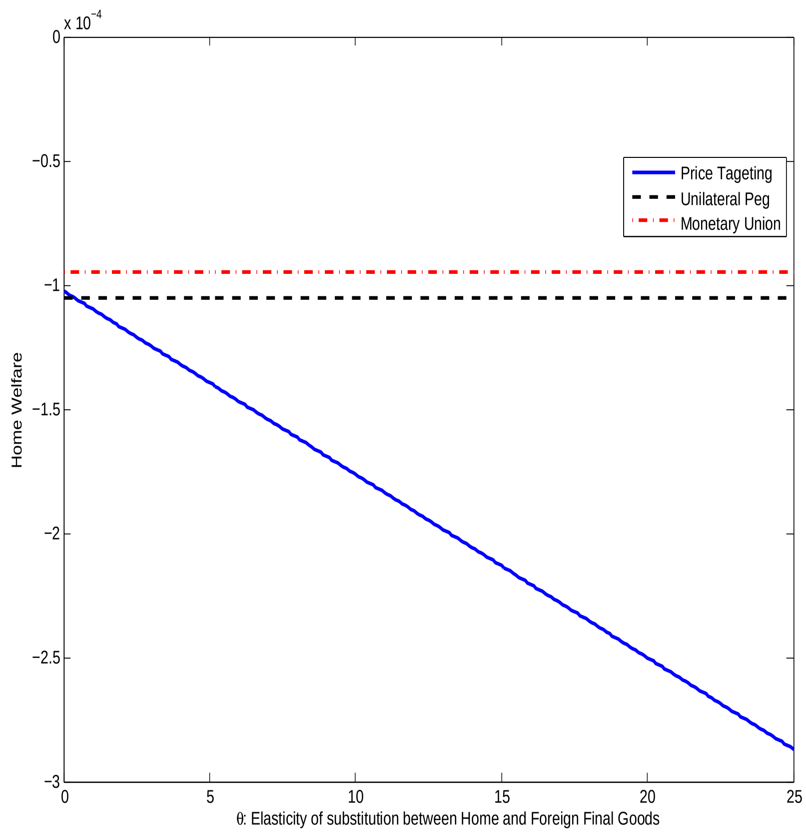


Figure 4: Home Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

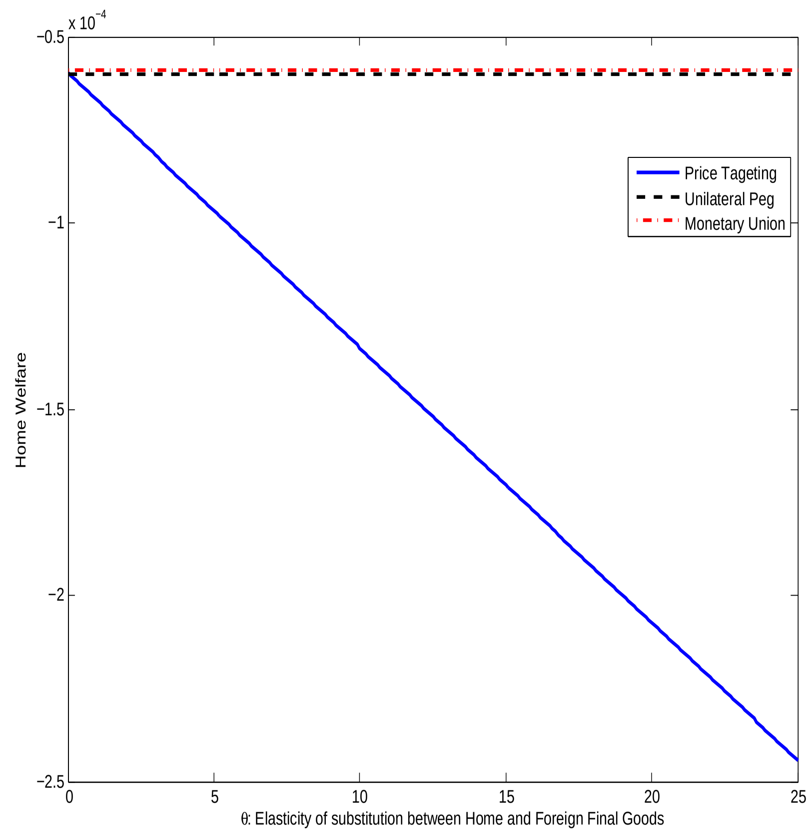


Figure 5: Home Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

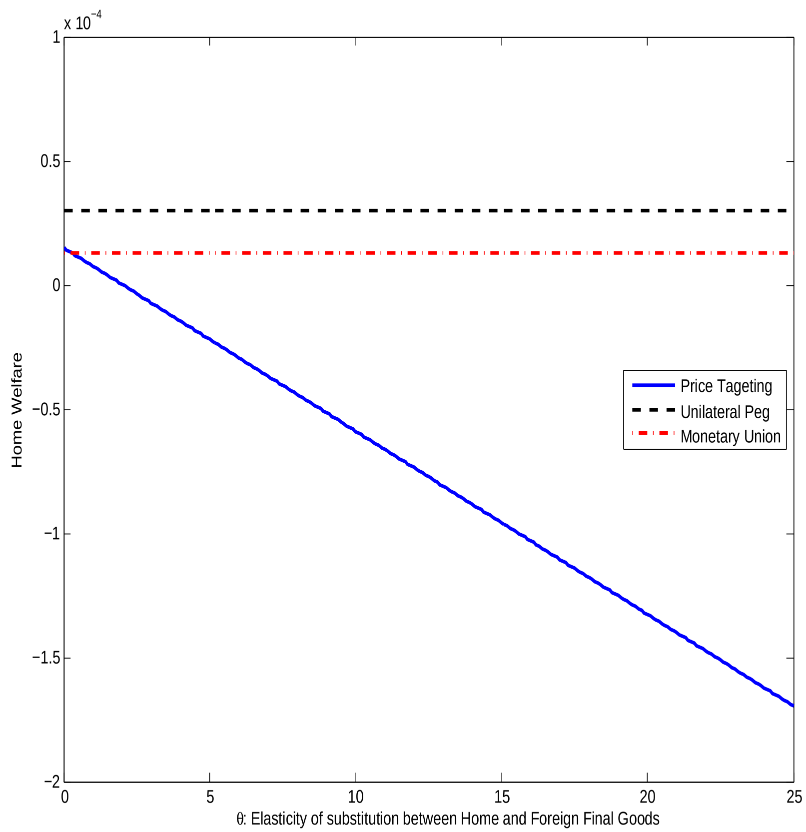


Figure 6: Home Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

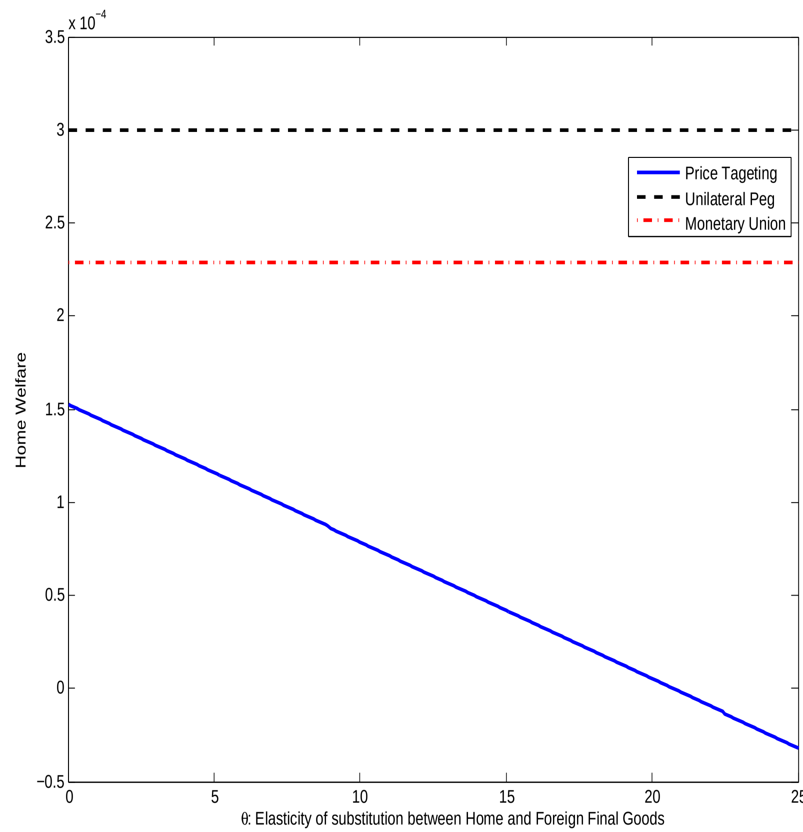


Figure 7: Home Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

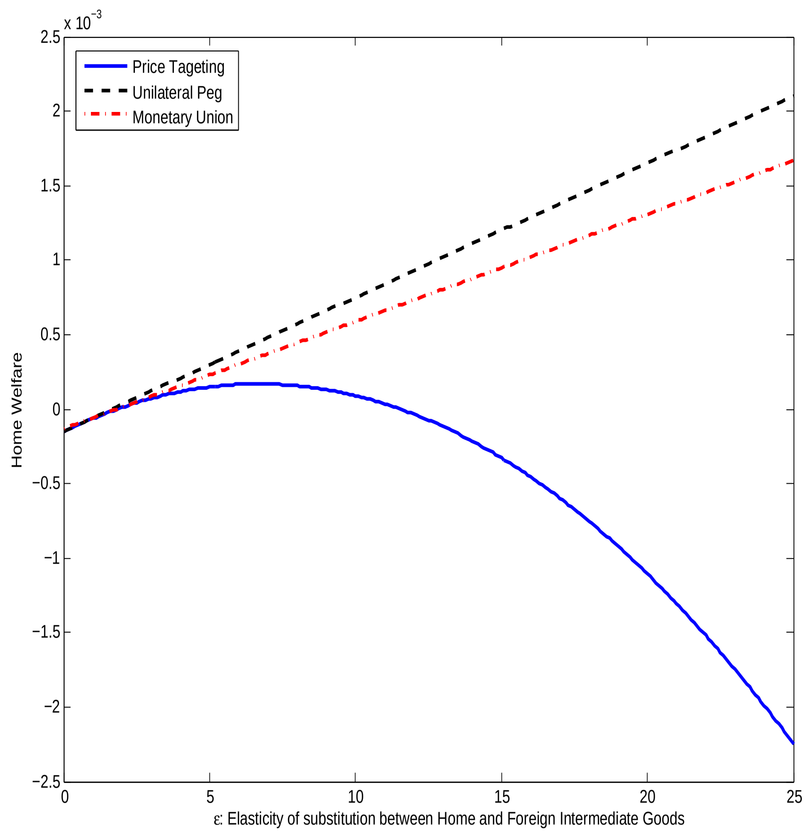


Figure 8: Home Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

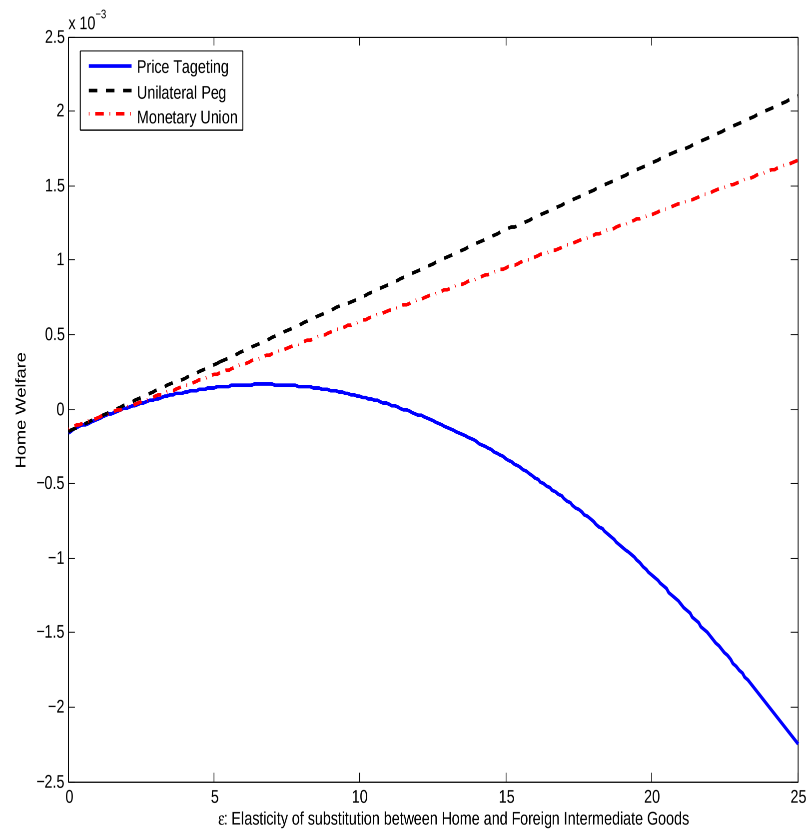


Figure 9: Home Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

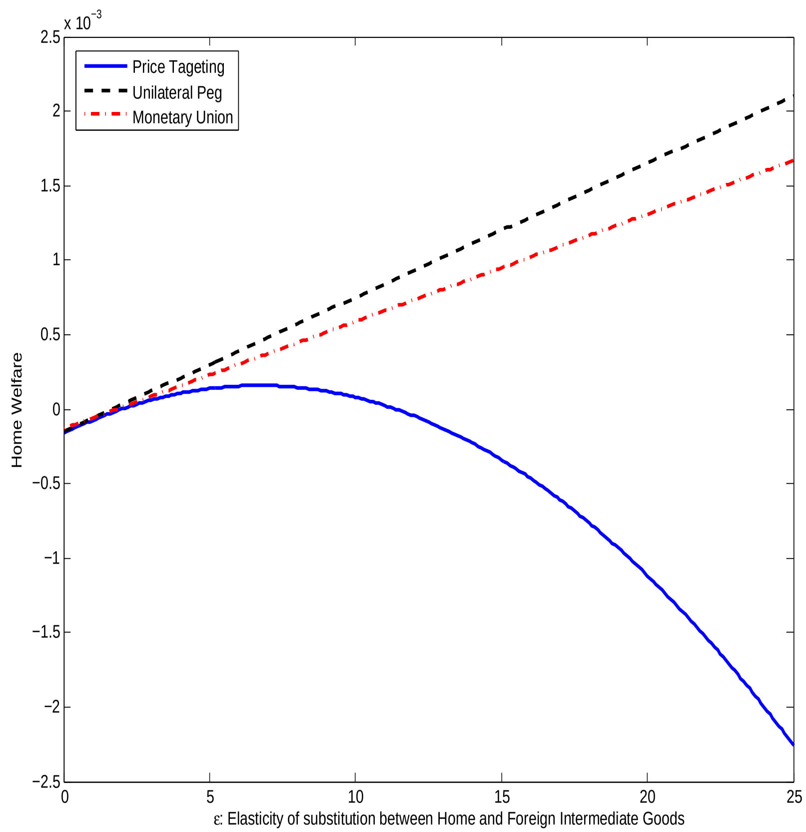


Figure 10: Home Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$

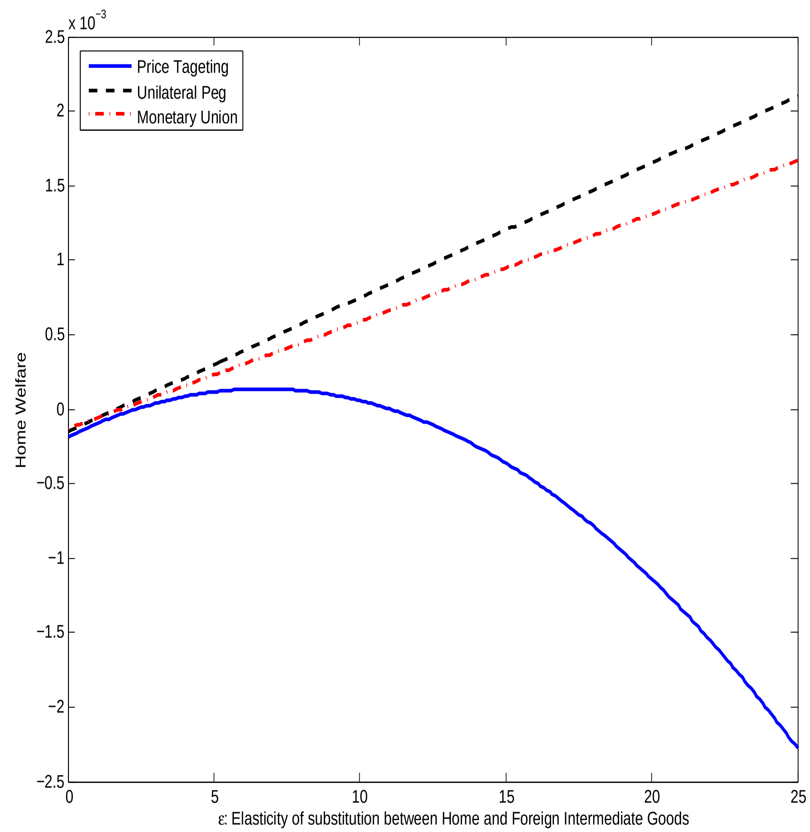


Figure 11: Foreign Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

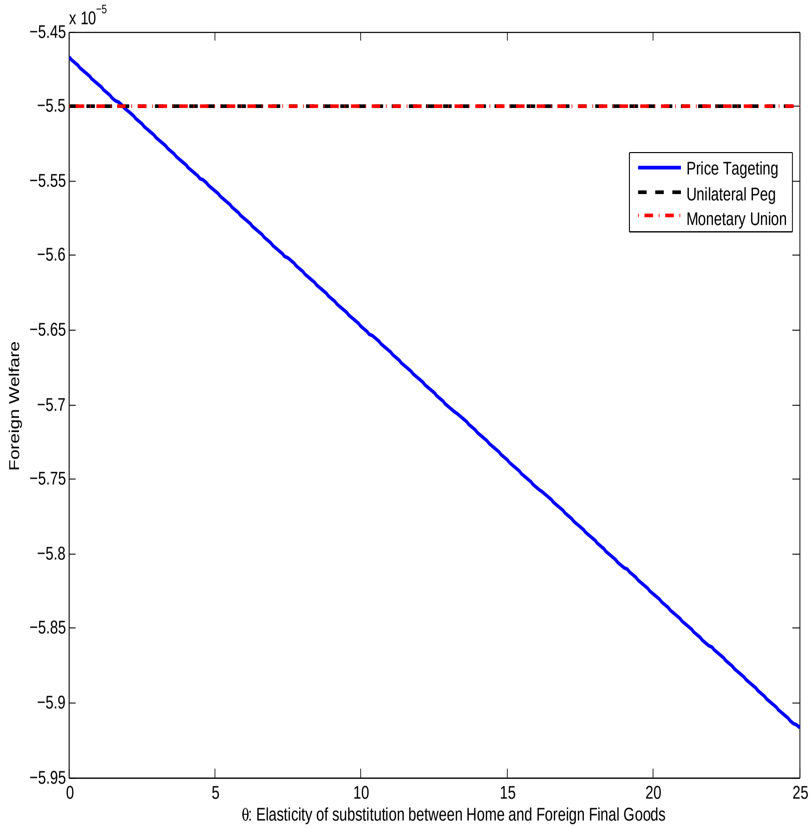


Figure 12: Foreign Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

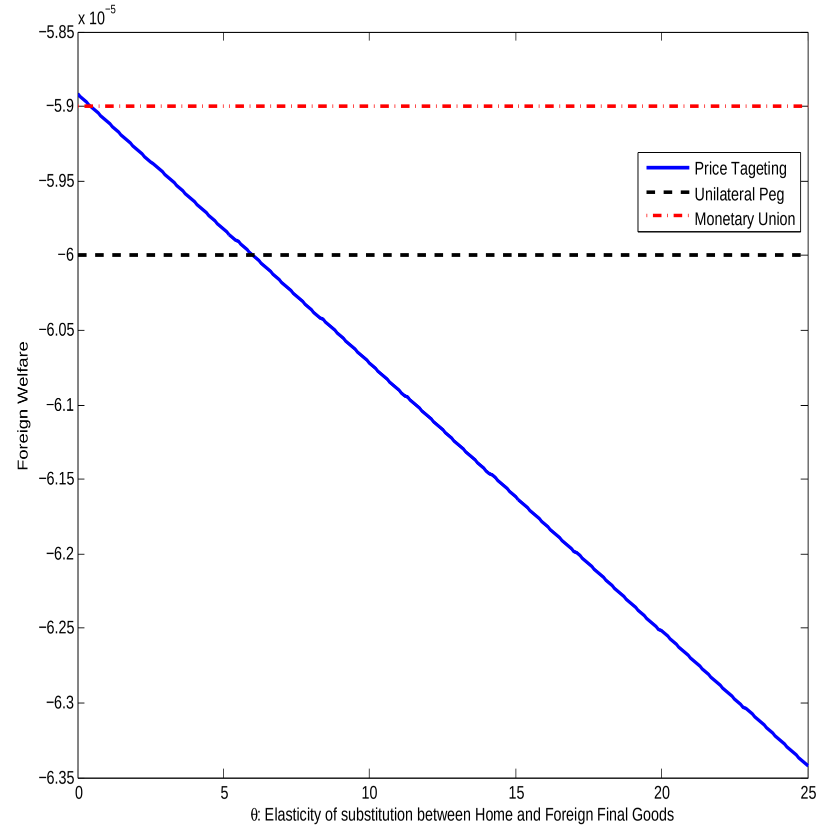


Figure 13: Foreign Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

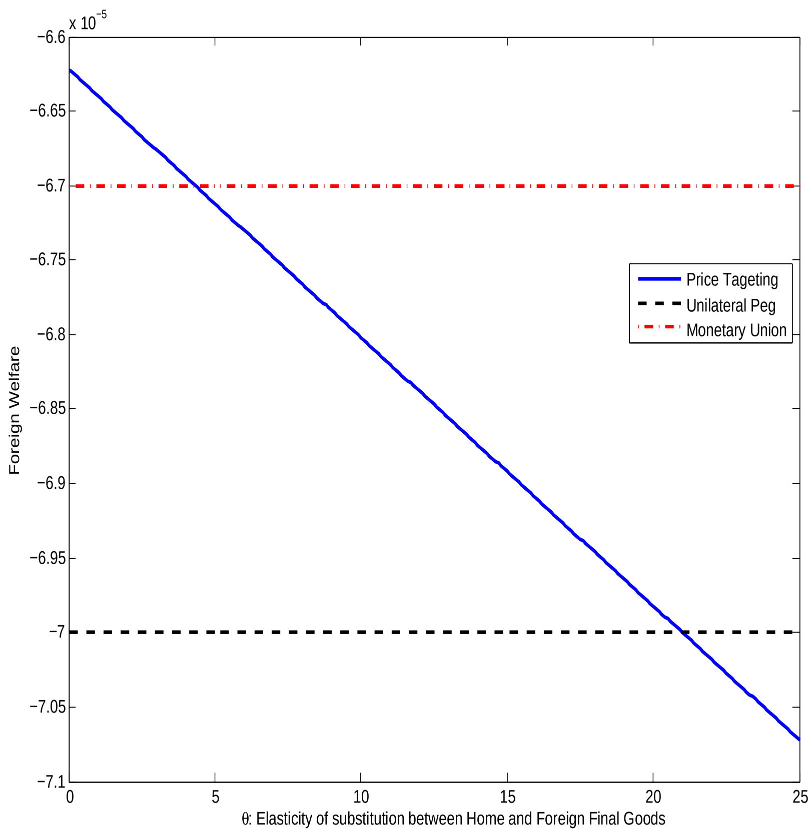


Figure 14: Foreign Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

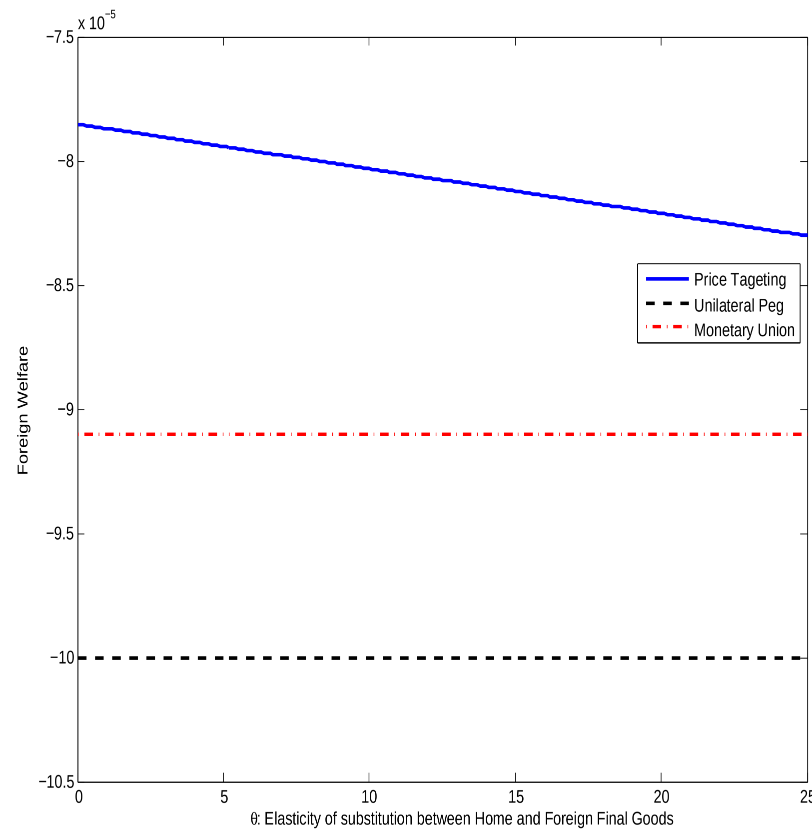


Figure 15: Foreign Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

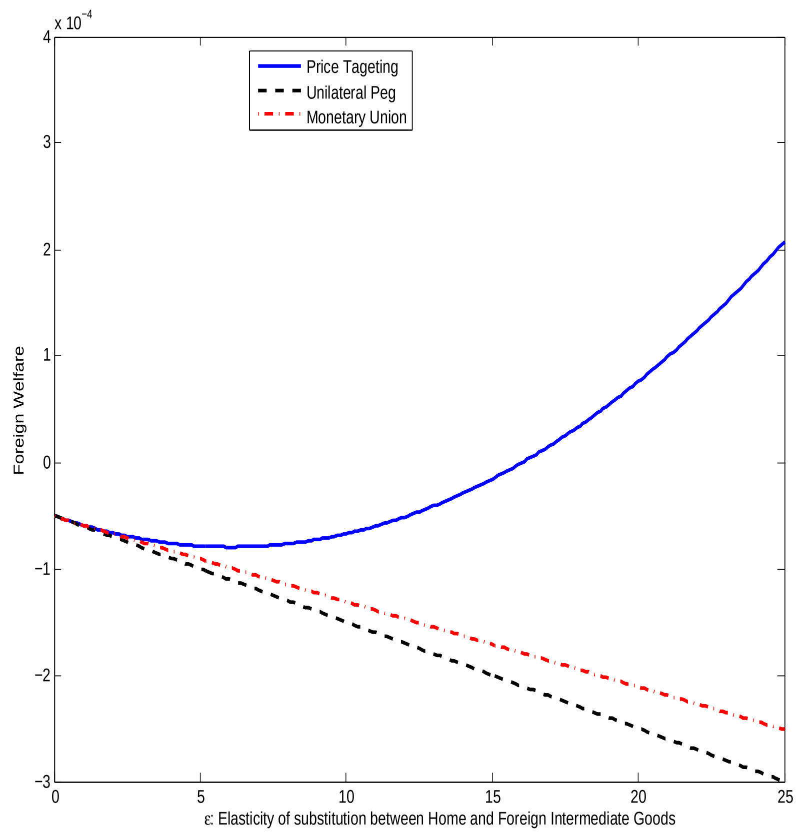


Figure 16: Foreign Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

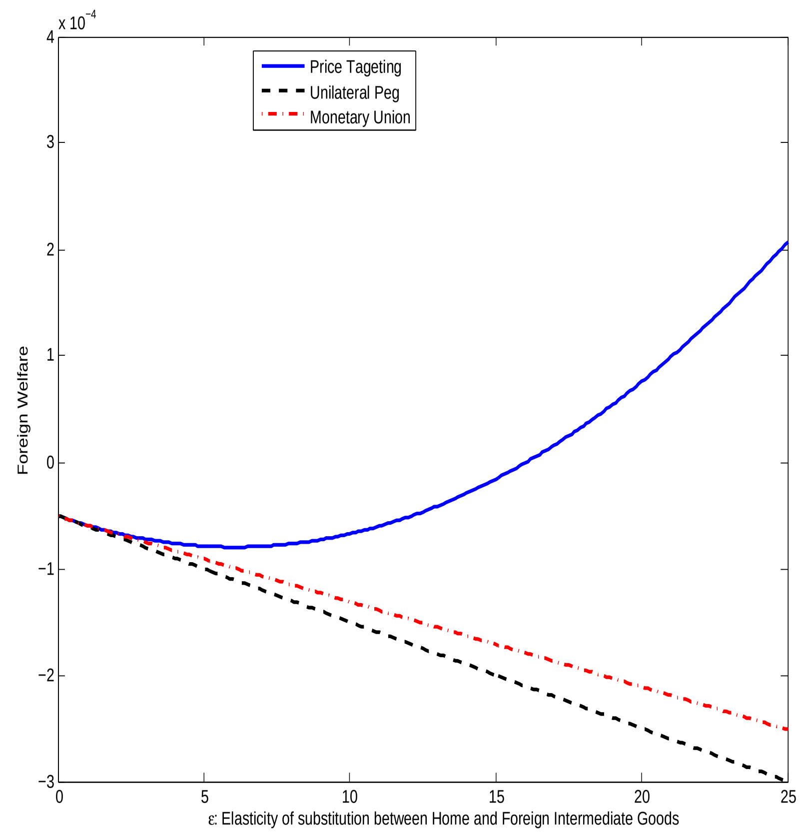


Figure 17: Foreign Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

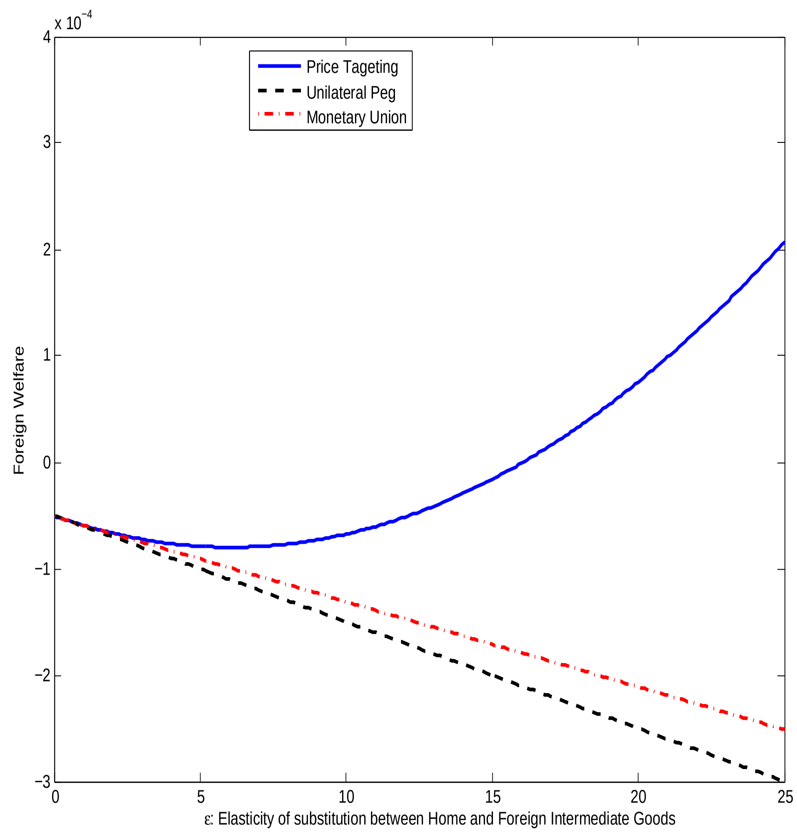
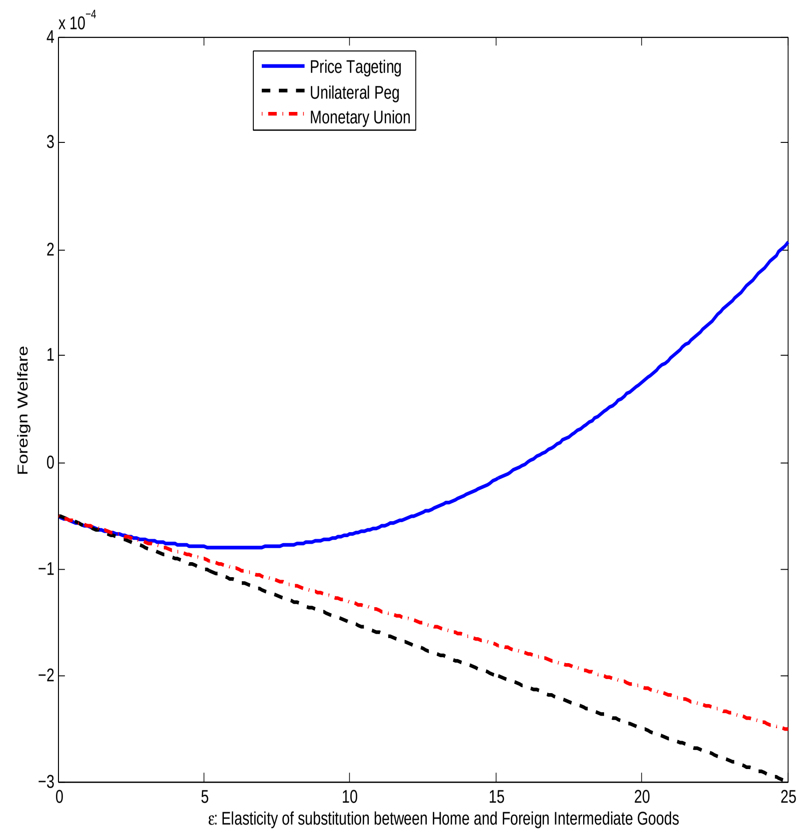


Figure 18: Foreign Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$



Technical Appendix of "Vertical Trade, Exchange Rate Pass-Through, and Exchange Rate Regime"

September 2012

(Not to be Published)

A.1 Log Linearization

The system consists of equations (5), (13)-(15), (19), (30), and their foreign counterparts, as well as (21)-(23), (27)-(29), and (37)-(38). Some of the equilibrium conditions are log-linear in themselves, such as the money demand equations, the labor supply equations, the risk-sharing condition, and the monetary policy rules.

$$\hat{M} = \hat{P} + \hat{C} \tag{A.1}$$

$$\hat{M}^* = \hat{P}^* + \hat{C}^* \tag{A.2}$$

$$\hat{W} = \hat{P} + \hat{C} + \psi \hat{L} \tag{A.3}$$

$$\hat{W}^* = \hat{P}^* + \hat{C}^* + \psi \hat{L}^* \tag{A.4}$$

$$\hat{P} + \hat{C} = \hat{S} + \hat{P}^* + \hat{C}^* \tag{A.5}$$

$$\hat{M} = -\delta_s \hat{S} - \delta_p \hat{P}^X \tag{A.6}$$

$$\hat{M}^* = -\delta_s^* \hat{S} - \delta_p^* \hat{P}^{*X} \tag{A.7}$$

First-order approximations of the other equilibrium conditions are given by

$$\hat{P} = n\hat{P}_H + (1-n)(\hat{P}_F^* + \hat{S}) + O(\epsilon^2) \tag{A.8}$$

$$\hat{P}^* = n\hat{P}_H^* + (1-n)\hat{P}_F^* + O(\epsilon^2) \tag{A.9}$$

$$\hat{\Lambda} = -\hat{A} + n\hat{\tilde{P}}_H + (1-n)(\hat{\tilde{P}}_F^* + \hat{S}) + O(\epsilon^2) \tag{A.10}$$

$$\hat{\Lambda}^* = -\hat{A}^* + n\hat{\tilde{P}}^*_H + (1-n)\hat{\tilde{P}}^*_F + O(\epsilon^2) \tag{A.11}$$

$$\hat{P}_H = \hat{P}^*_H = \hat{P}^*_F = \hat{\tilde{P}}_H = \hat{\tilde{P}}^*_H = \hat{\tilde{P}}^*_F = 0 + O(\epsilon^2) \tag{A.12}$$

$$\begin{aligned} \hat{L} &= -\hat{\tilde{A}} - n\left[\epsilon(\hat{\tilde{P}}_H - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] \\ &\quad -(1-n)\left[\epsilon(\hat{\tilde{P}}^*_H - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C}^* + O(\epsilon^2) \end{aligned} \tag{A.13}$$

$$\begin{aligned} \hat{L}^* &= -\hat{\tilde{A}}^* - n\left[\epsilon(\hat{\tilde{P}}^*_F + \hat{S} - \hat{\Lambda}) + (1-\epsilon)\hat{A}\right] \\ &\quad -(1-n)\left[\epsilon(\hat{\tilde{P}}^*_F - \hat{\Lambda}^*) + (1-\epsilon)\hat{A}^*\right] + n\hat{C} + (1-n)\hat{C}^* + O(\epsilon^2) \end{aligned} \tag{A.14}$$

A.2 Figures for Section 4.3

Figure A.1 - Figure A.16 repeat the exercises in the paper by shutting down the final goods sector. In other words, Figure A.1 - Figure A.16 correspond to Figure (4) - (19) in the text but with $\sigma^2_A = \sigma^2_{A^*} = 0$ and $\sigma^2_{\tilde{A}} = \sigma^2_{\tilde{A}^*} = 0.0001$.

A.3 Final Goods PPI Inflation Targeting

Consider the following inflation targeting regime, in which both home and foreign central banks target the PPI of final goods, i.e., $P^X = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda$, $P^{*X} = \frac{\phi}{(\phi-1)(1+\gamma)}\Lambda^*$, $\delta_s = \delta^*_s = 0$, and $\delta_p = \delta^*_p \to \infty$. In this case, exchange rate will respond only to the final goods sector's productivity shocks and will create inefficient spillovers from the final goods sector to the intermediate goods sector. It first appears that the final goods PPI targeting works just like the intermediate goods PPI targeting where the welfare costs originate from the vertical chain of production and trade. However, these costs are much larger, indeed unbounded, when it is the intermediate goods sector that faces the negative spillovers. To explain this, let us look at the linearized model.

From equations (A.10)-(A.12), we have

$$\hat{\Lambda} = -\hat{A} + (1-n)\hat{S} + O(\epsilon^2) \tag{A.15}$$

$$\hat{\Lambda}^* = -\hat{A}^* + O(\epsilon^2) \tag{A.16}$$

To replicate the flexible prices of final goods, strict final goods PPI inflation targeting requires

$$\hat{M} = -\delta_p[-\hat{A} + (1-n)\hat{S}] + O(\epsilon^2) \tag{A.17}$$

$$\hat{M}^* = -\delta_p^*(-\hat{A}^*) + O(\epsilon^2) \tag{A.18}$$

Moreover, from equations (A.1)-(A.4), we know

$$\hat{W} = \hat{M} \tag{A.19}$$

$$\hat{W}^* = \hat{M}^* \tag{A.20}$$

That is to say, wages in each country will respond infinitely to the final goods sector's productivity shocks, which is inefficient since labor is only used in the intermediate goods sector not in the final goods sector. Such enormous fluctuations in wages, as well as in the unit costs of producing intermediate goods, create huge uncertainties in the economy. Labor supply becomes much more volatile, and intermediate goods producers ask for much higher risk premiums when they pre-set the prices. As a result, households are expected to consume less, work more, and endure massive volatilities in labor supply. Welfare for both countries are a lot lower than those under the final goods PPI inflation targeting. In fact, the stricter the final goods PPI inflation targeting (i.e., the higher δ_p and δ_p^* are), the lower the welfare for each country. As $\delta_p = \delta_p^* \to \infty$, the welfare for both countries approach negative infinity. This is why we choose not to focus on this type of policy regime in the paper.

Figure A.1: Home Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

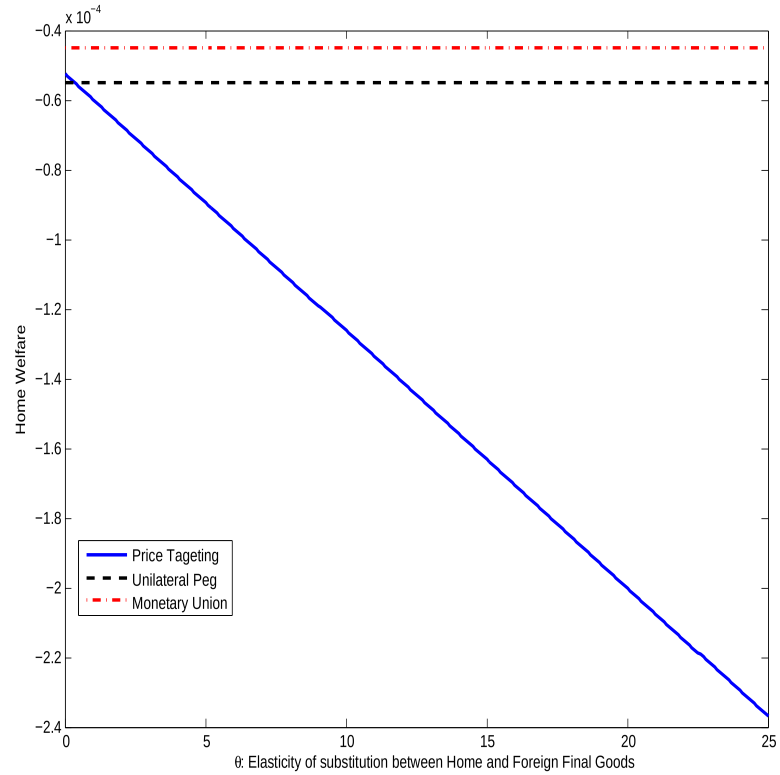


Figure A.2: Home Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

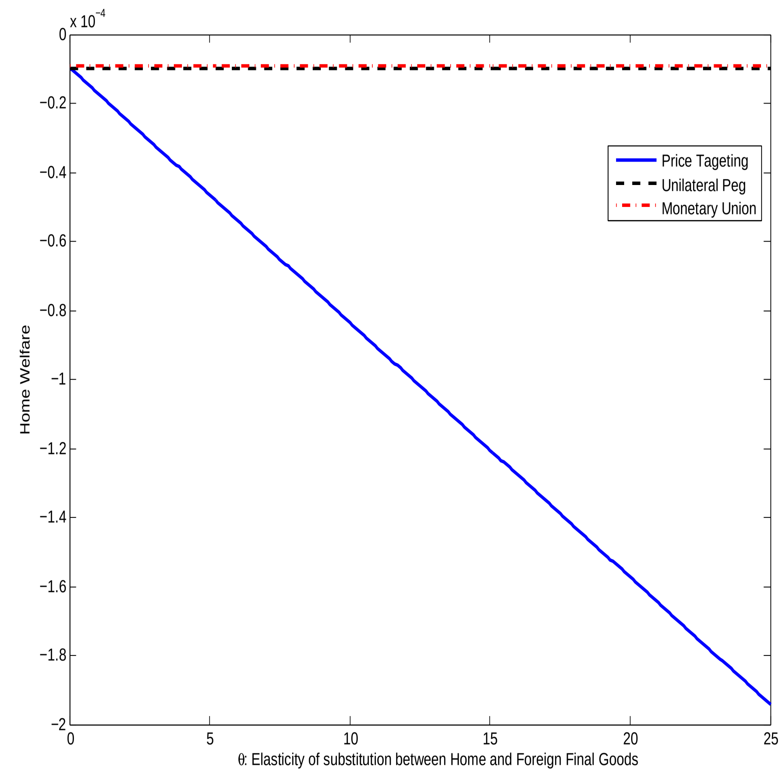


Figure A.3: Home Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

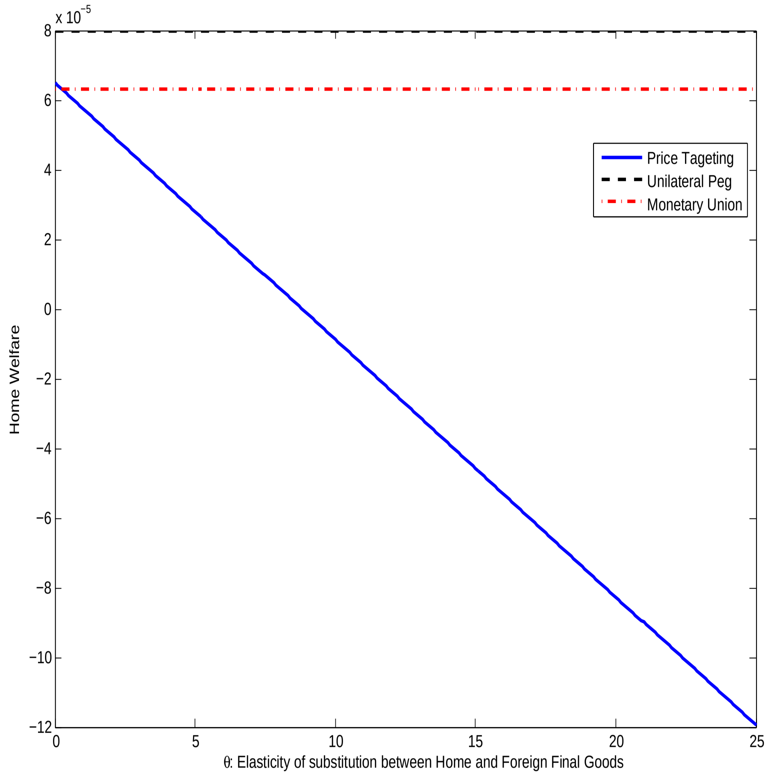


Figure A.4: Home Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

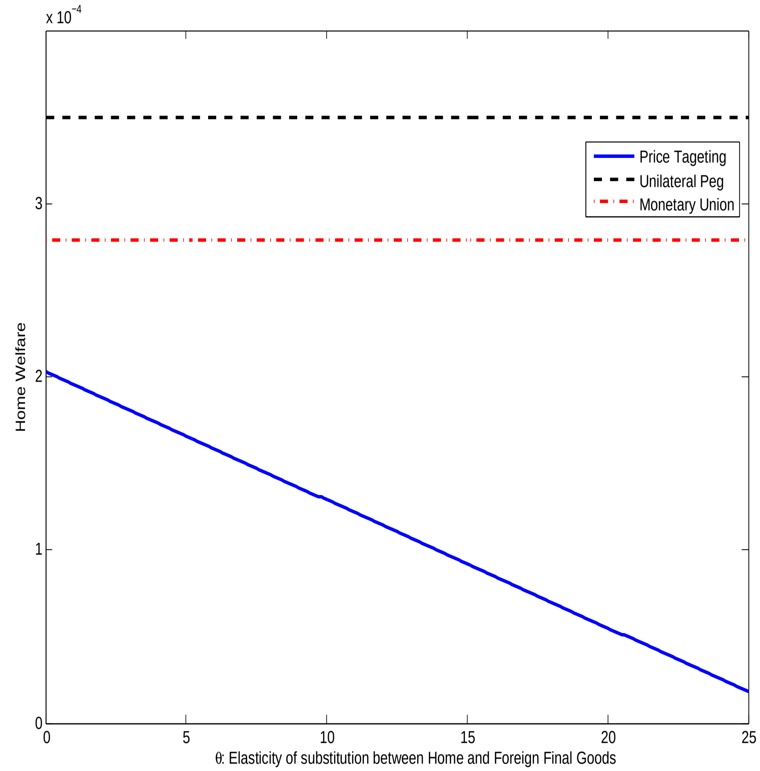


Figure A.5: Home Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

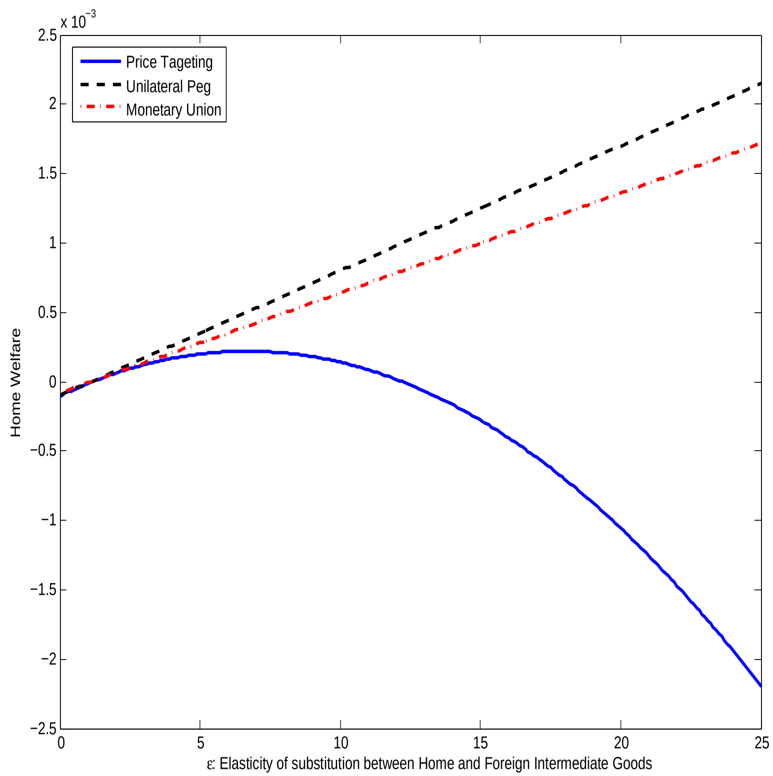


Figure A.6: Home Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

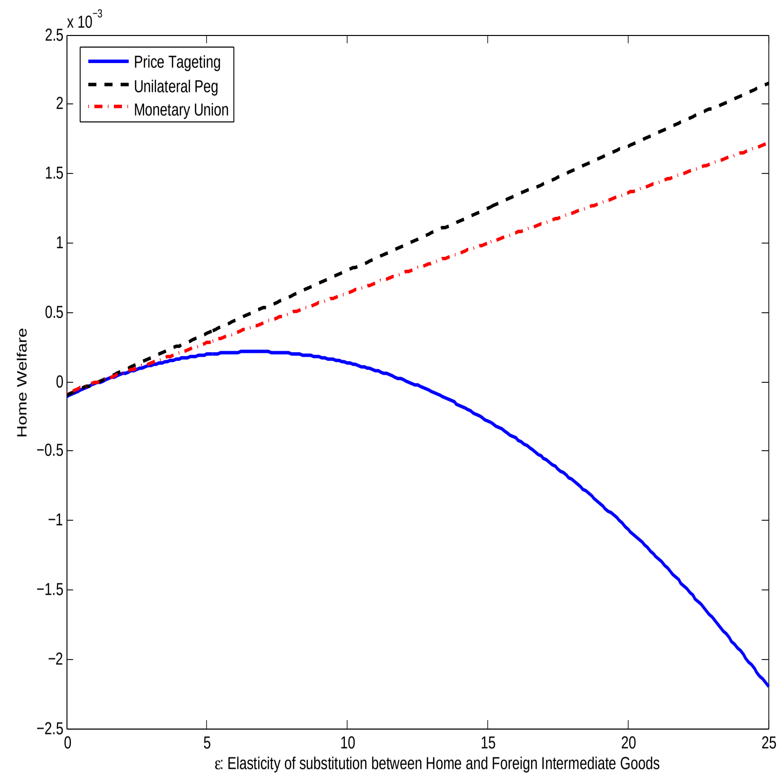


Figure A.7: Home Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

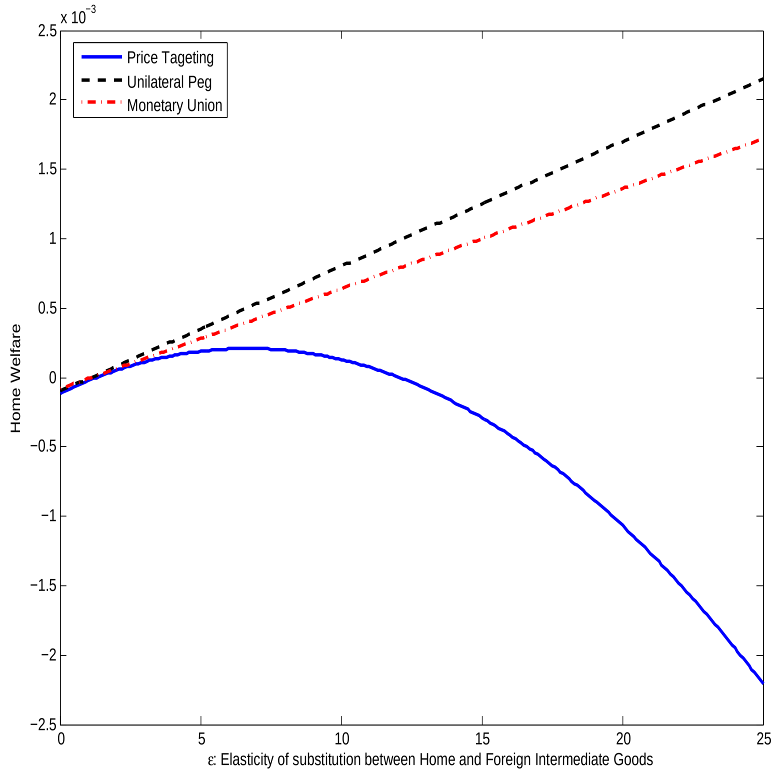


Figure A.8: Home Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$

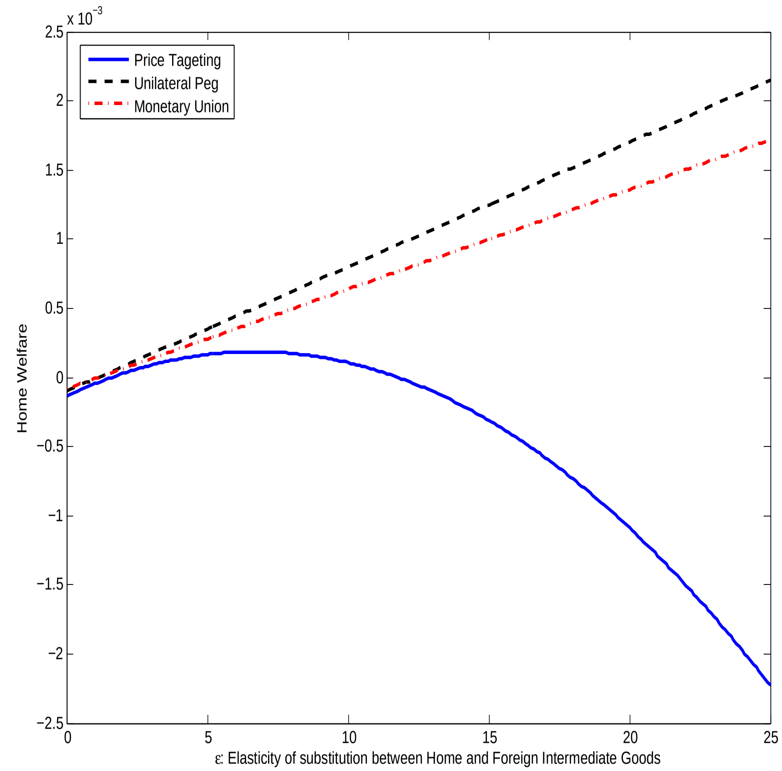


Figure A.9: Foreign Welfare Comparison: $\epsilon = 0.5$, $\theta \in [0, 25]$

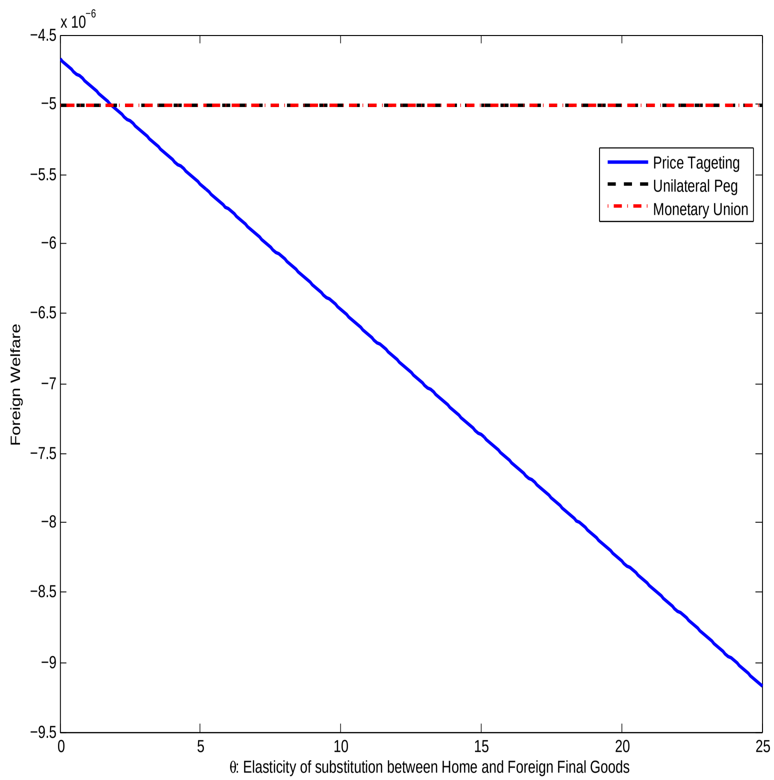


Figure A.10: Foreign Welfare Comparison: $\epsilon = 1$, $\theta \in [0, 25]$

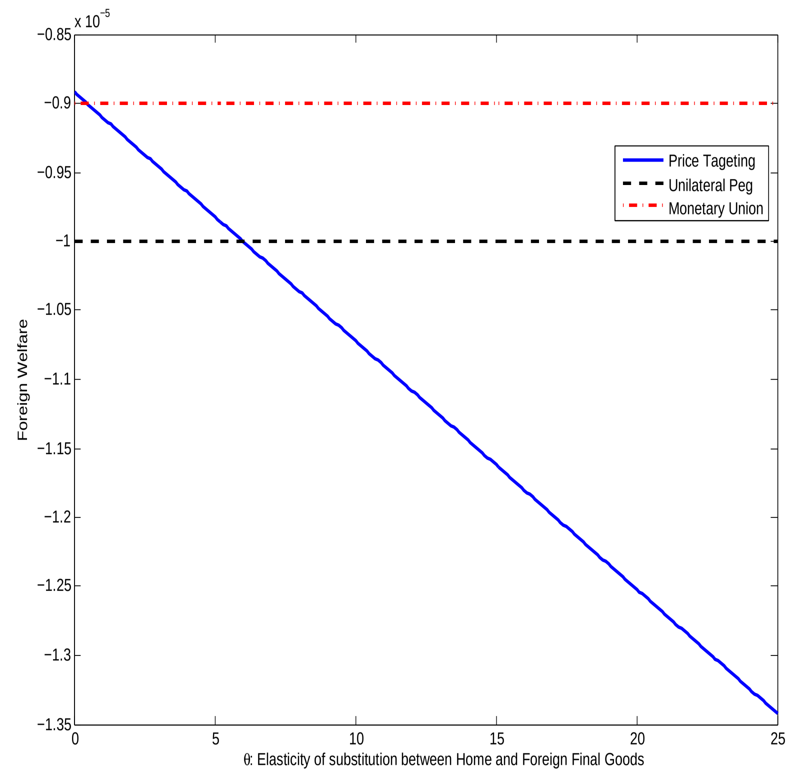


Figure A.11: Foreign Welfare Comparison: $\epsilon = 2$, $\theta \in [0, 25]$

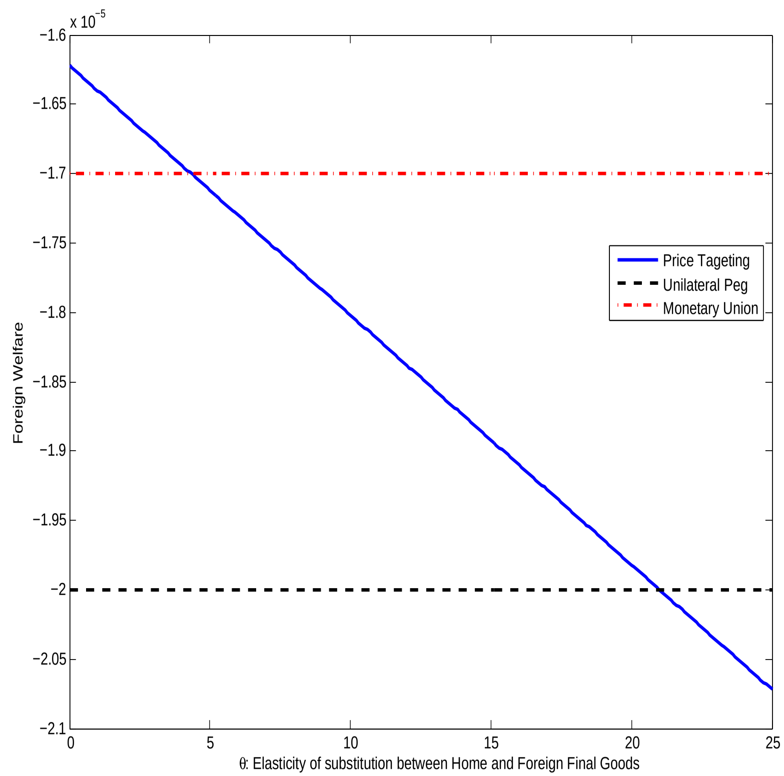


Figure A.12: Foreign Welfare Comparison: $\epsilon = 5$, $\theta \in [0, 25]$

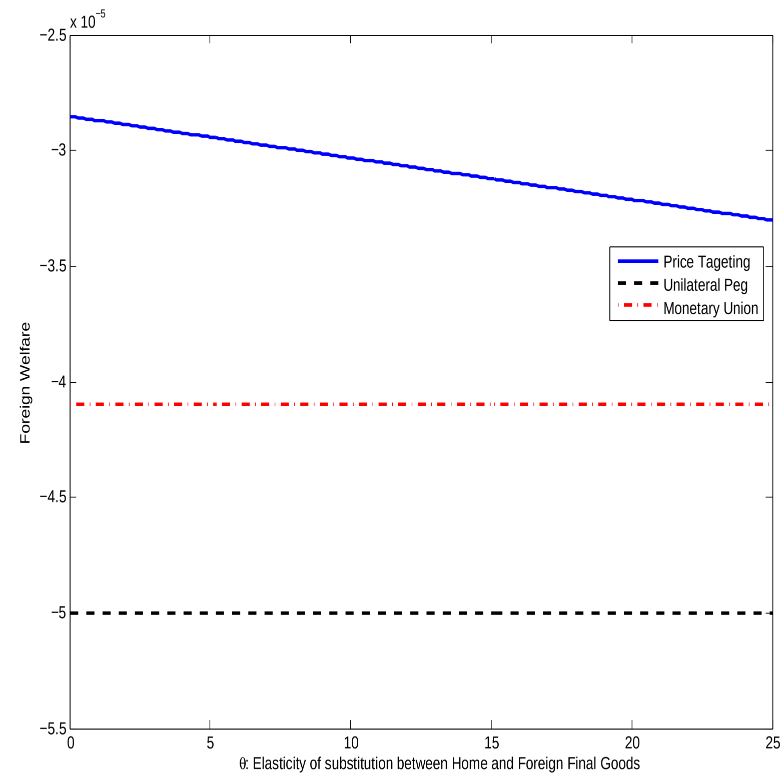


Figure A.13: Foreign Welfare Comparison: $\theta = 0.5$, $\epsilon \in [0, 25]$

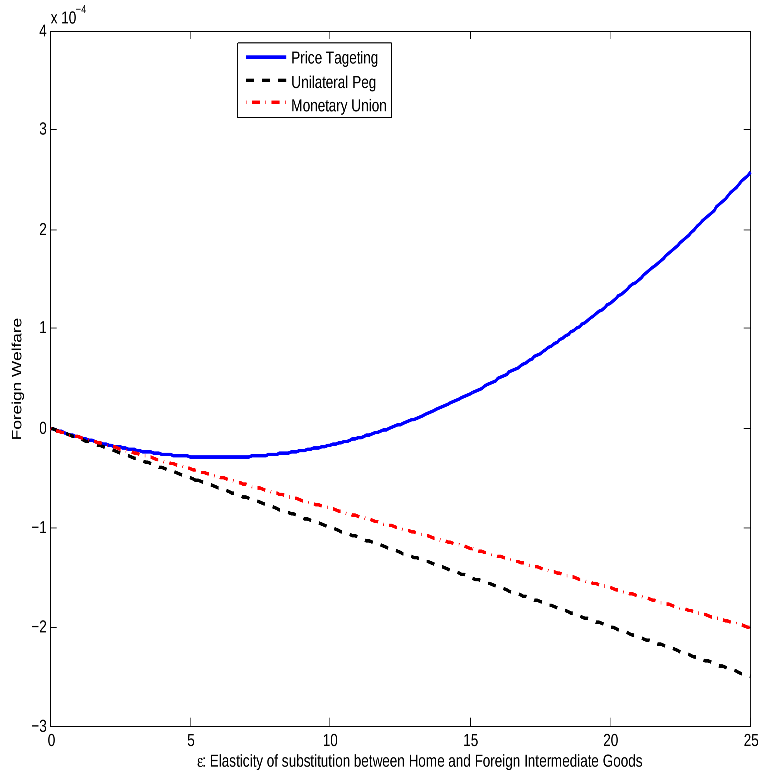


Figure A.14: Foreign Welfare Comparison: $\theta = 1$, $\epsilon \in [0, 25]$

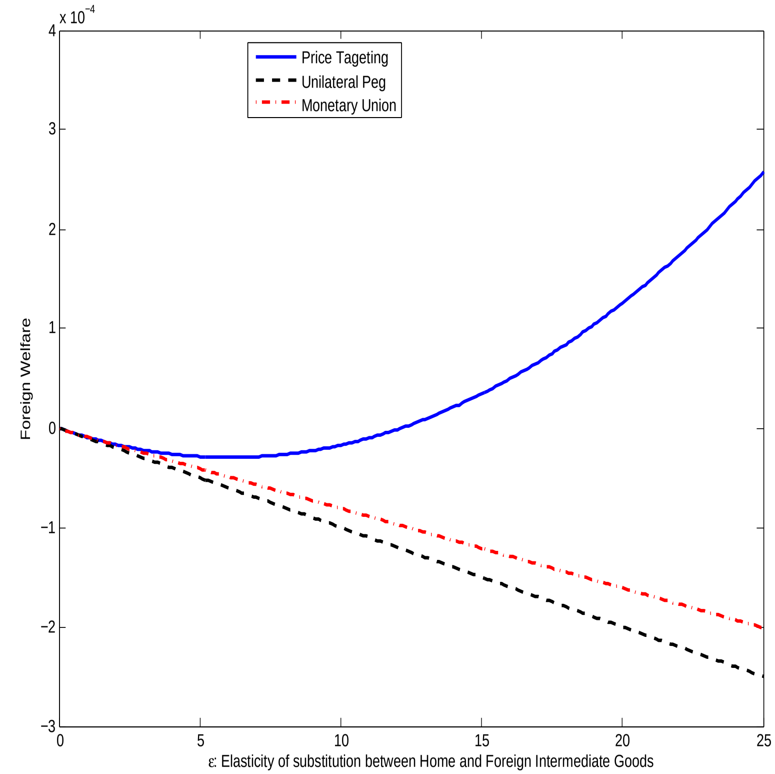


Figure A.15: Foreign Welfare Comparison: $\theta = 2$, $\epsilon \in [0, 25]$

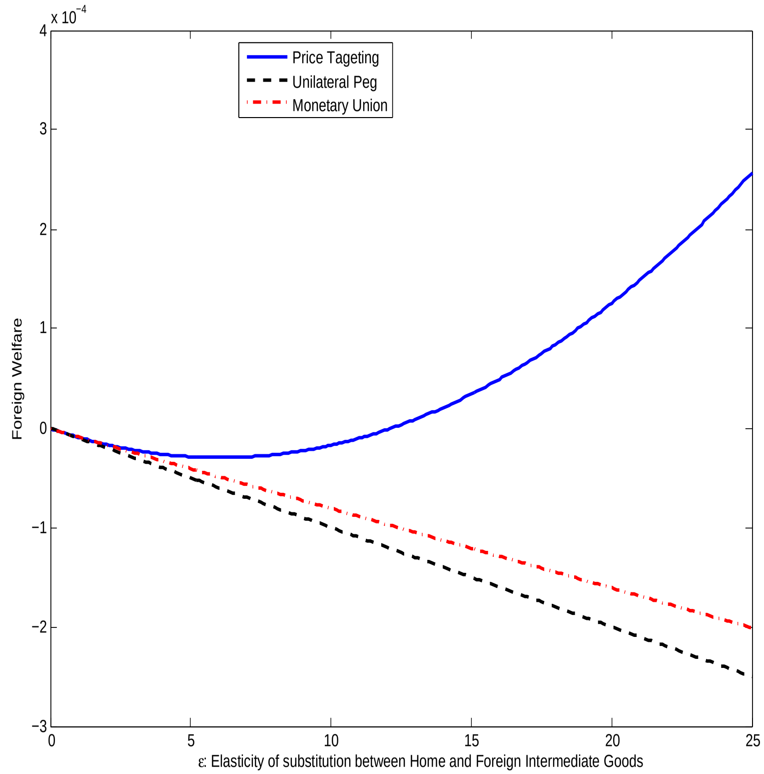


Figure A.16: Foreign Welfare Comparison: $\theta = 5$, $\epsilon \in [0, 25]$